

CANONICAL QUANTIZATION OF $N=1$ SUPERGRAVITY WITH SUPERMATTER: THE GENERAL CASE AND A FRW MODEL

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Received 18 August 1994

The general theory of $N = 1$ supergravity with supermatter using a canonical approach is described. The supersymmetry and gauge constraint generators are presented. The framework is applied to the study of a Friedmann minisuperspace model. We consider a Friedmann $k = +1$ geometry and a family of spin-0 as well as spin-1 gauge fields together with their odd (anti-commuting) spin-1/2 partners. When both the spin-1 field and its fermionic partner are set equal to zero, the physical states of our simplified model correspond effectively to those of a mini-superspace quantum cosmological model possessing $N=4$ local supersymmetry coupled to complex scalars with spin-1/2 partners. The different supermatter models correspond to different Kähler metrics. For the cases of spherically symmetric and flat Kähler geometries the general solution for the quantum state have a simple form. However, although they allow a Hartle-Hawking solution, they do not permit a wormhole state.

PACS numbers: 04.60.+ n, 04.65.+ e, 98.80. Hw

I. Introduction

In the last ten years or so, the subjects of supersymmetric quantum gravity and cosmology have achieved a considerable number of very interesting results and conclusions. Several approaches may be found in the literature, namely the triad ADM canonical formulation [1–13], the σ -model supersymmetric extension in quantum cosmology [14–16] and another approach based on Ashtekar variables [17–20]. A detailed review on this subject is currently in preparation [21].

The canonical formulation framework of $N=1$ (pure) supergravity was presented in Ref. [1]. It can be shown that it is sufficient, in finding a physical state, to solve the Lorentz and supersymmetry constraints of the theory because the algebra of constraints of the theory leads to anti-commutation relations implying that a physical wave functional Ψ will also obey the Hamiltonian constraints [1,22].

Using the triad ADM canonical formulation, the Bianchi-I model in $N=1$ supergravity with no cosmological constant ($\Lambda=0$) was considered in Ref. [2] and the quantum states are in the bosonic and filled fermionic sectors and are of the form $\exp(-\frac{1}{2}h^{-1/2})$, where $h = \det h_{ij}$ is the determinant of the three-metric [10]. In the case of Bianchi IX with $\Lambda = 0$, there are two states, of the form $\exp(\pm I/\hbar)$ where I is a certain Euclidean action, one in the empty and one in the filled fermionic sector [3,15]. When the usual choice of spinors constant in the standard basis is made for the gravitino field, the bosonic state

$\exp(-I/\hbar)$ is the wormhole state [3,24]. With a different choice, one obtains the Hartle-Hawking state [23,25]. Similar states were found for $N = 1$ supergravity in the more general Bianchi models of class A [26]. [Supersymmetry (as well as other considerations) forbids mini-superspace models of class B .] The extension of this analysis to the simple case where a cosmological constant is present in $N=1$ supergravity is described in Refs.[8–10]. It was found by imposing the supersymmetry and Lorentz constraints that there are then *no* physical states in the models we have considered. Regarding the $k=1$ Friedmann-Robertson-Walker model, where the fermionic degrees of freedom of the gravitino field are very restricted, we have found a bosonic quantum physical state, namely the Hartle-Hawking state for a De Sitter solution. If one studies generic cosmological models using perturbation theory about the $k = +1$ Friedmann model, it seems that the gravitational and gravitino modes that are allowed to be excited in a supersymmetric Bianchi-IX model contribute in such a way as to forbid any physical solutions of the quantum constraints. This suggests that in a complete perturbation expansion we would have to conclude that the full theory of $N=1$ supergravity with a non-zero cosmological constant should have *no* physical states.

One would like to extend this understanding to more general supergravity models involving lower-spin fields. One possibility is to consider higher- N gauged supergravity models [34], but these are technically difficult in the approach used in [11] because they contain

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a Λ -term which breaks chirality. Instead, we study here the model of $N=1$ supergravity coupled to supermatter [27], and in particular its supersymmetry constraints, especially in the case with zero analytic potential $P(\Phi^I)$. Such a study was performed in [13] for the case of $N=1$ supergravity coupled to supermatter [27].

The study of 1-dimensional mini-superspace models with local supersymmetry, based on this, leads to further understanding of quantum cosmology and gravity. Clearly, a richer and more interesting class of minisuperspace models is given by coupling supermatter to $N=1$ supergravity in 4 dimensions, and then reducing the model to 1 dimension by making a suitable homogeneous Ansatz [4–7]. In particular, from (1+3) dimensional $N=1$ supergravity a dimensional reduction allows one to obtain a (1+0)-dimensional theory with $N=4$ supersymmetry.

In Refs.[4–7] an Ansatz for the gravitational and spin-3/2 fields was introduced in order to reduce pure $N=1$ supergravity in 4 dimensions to a locally supersymmetric quantum cosmological model in 1 dimension, assuming a Friedmann $k = +1$ geometry and homogeneity of the spin-3/2 field on the S^3 spatial sections. The Hamiltonian structure of the resulting theory was found, leading to the quantum constraint equations. The general solution to the quantum constraints is very simple in this case, and the Hartle–Hawking wave-function can be found. A more general model was also studied, in which $N=1$ supergravity is coupled to locally supersymmetric matter, there taken to be a massive complex scalar with spin-1/2 partner. In the massless case, the general solution of the quantum constraints can be found as an integral expression, including a ground quantum wormhole state [6].

Here we describe the extension of mini-superspace quantum cosmological models when $N=1$ supergravity is coupled to locally supersymmetric matter [4–7,13]. We consider the more general supergravity theory with a Friedmann $k=+1$ geometry and a family of spin-0 as well as spin-1 gauge fields together with their odd (anti-commuting) spin- $\frac{1}{2}$ partners [13]. The general such theory is described in detail in Ref.[27] (the minisuperspace models with supermatter described in Refs.[4–7] followed a four-dimensional model of Das et al [28]). Our Ansätze for the fields are such as to reduce the $N=1$ supergravity plus supermatter in 4 dimensions [13,27] to a locally supersymmetric $N=4$ FRW quantum cosmological model in 1 dimension. Hence, we assume a Friedmann $k = +1$ geometry, and the other fields are chosen as to respect the homogeneity and isotropy of the S^3 spatial sections. The choice made for the spin-1 field is described in Refs.[29–31] and for the other fields the details are given in Refs.[4,5,7]. The quantum supersymmetry constraints give rise to a set of first-order coupled partial differential equations for the components of the wave function. As an intermediate stage in our re-

search project, we put both the spin-1 field and its fermionic partner equal to zero. The physical states of our simplified model correspond effectively to those of a mini-superspace quantum cosmological model possessing $N=4$ local supersymmetry coupled to complex scalars with spin-1/2 partners. The different supermatter models are given by specifying a Kähler metric for the scalar fields; the allowed quantum states then depend on the Kähler geometries. For the cases of spherically symmetric and flat Kähler geometries we have found the general solution for the quantum state with a very simple form. However, these states are somewhat different from the ones presented in Refs. [4–7]; although they allow a Hartle-Hawking solution, we cannot find a wormhole state.

This paper is organized as follows. In section II the more general theory of $N=1$ supergravity with supermatter is described using a canonical approach (see Ref. [13]). The supersymmetry and gauge constraint generators are also found. In section III we specify our Ansätze for the the gravitational and spin-3/2 fields as well as for the supermatter fields and their fermionic partners. The supersymmetry constraints are derived from the reduced action in section IV. In section V we solve the quantum constraints and find a general solution for the quantum state of the universe. We also make some comments on the issue of determining the operator ordering in the constraints. A discussion and interpretation of our results is presented in section VI, together with a summary of our research and indications of further possibilities.

II. Canonical Formulation of $N=1$ Supergravity with Supermatter

The Lagrangian of the more general gauged supergravity theory coupled to a family of spin-0 as well as spin-1 fields together with their odd (anti-commuting) spin- $\frac{1}{2}$ partners is given in Eq. (25.12) of Ref.[27]; it is too long to write out here. It depends on the tetrad $e^{AA'}{}_{\mu}$, where A, A' are two-component spinor indices using the conventions of [1] and μ is a space-time index, the odd (anti-commuting) gravitino field $(\psi^A{}_{\mu}, \tilde{\psi}^{A'}{}_{\mu})$, a vector field $A_{\mu}^{(a)}$ labelled by an index (a) , its odd spin- $\frac{1}{2}$ partners $(\lambda_A^{(a)}, \tilde{\lambda}_{A'}^{(a)})$, a family of scalars (Φ^I, Φ^{J*}) and their odd spin- $\frac{1}{2}$ partners $(\chi_A^I, \tilde{\chi}_{A'}^{J*})$. The indices I, \dots, J^*, \dots are Kähler indices, and there is a Kähler metric

$$g_{IJ^*} = K_{IJ^*} \quad (2.1)$$

on the space of (Φ^I, Φ^{J*}) , where K_{IJ^*} is a shorthand for $\partial^2 K / \partial \Phi^I \partial \Phi^{J^*}$ with K the Kähler potential. Each index (a) corresponds to an independent Killing vector field of the Kähler geometry. Such Killing vectors are holomorphic vector fields:

$$X^{(b)} = X^{I(b)}(\Phi^J) \frac{\partial}{\partial \Phi^I},$$

$$X^{*(b)} = X^{I*(b)} \left(\Phi^{J*} \right) \frac{\partial}{\partial \Phi^{I*}} . \quad (2.2)$$

Killing's equation implies that there exist real scalar functions $D^{(a)}(\Phi^I, \Phi^{J*})$ known as Killing potentials, such that

$$\begin{aligned} g_{IJ*} X^{J*(a)} &= i \frac{\partial}{\partial \Phi^I} D^{(a)} , \\ g_{IJ*} X^{I(a)} &= -i \frac{\partial}{\partial \Phi^{J*}} D^{(a)} . \end{aligned} \quad (2.3)$$

We shall consider instead the Hamiltonian formulation of the theory [13]. The Hamiltonian has the form

$$\begin{aligned} H &= N\mathcal{H}_\perp + N^i \mathcal{H}_i + \psi^A{}_0 S_A \\ &\quad + \tilde{S}_{A'} \tilde{\psi}^{A'}{}_0 + A_0^{(a)} Q_{(a)} \\ &\quad + M_{AB} J^{AB} + \tilde{M}_{A'B'} \tilde{J}^{A'B'} , \end{aligned} \quad (2.4)$$

expected for a theory with the corresponding gauge invariances. Here N and N^i are the lapse function and shift vector [1], while \mathcal{H}_\perp and \mathcal{H}_i are the (modified) generators of deformations in the normal and tangential directions. S_A and $\tilde{S}_{A'}$ are the local supersymmetry generators, $Q_{(a)}$ is the generator of gauge invariance, and J^{AB} and $\tilde{J}^{A'B'}$ are the generators of local Lorentz rotations, while M_{AB} and $\tilde{M}_{A'B'}$ are Lagrange multipliers giving the amount of Lorentz rotation applied per unit time. Classically, the constraints $\mathcal{H}_\perp, \mathcal{H}_i$, etc. vanish, and the set of (first-class) constraints forms an algebra.

Quantum-mechanically, the constraints become operators which annihilate physical states Ψ :

$$\begin{aligned} \mathcal{H}_\perp \Psi &= 0, & \mathcal{H}_i \Psi &= 0, \\ S_A \Psi &= 0, & \tilde{S}_{A'} \Psi &= 0, \\ Q_{(a)} \Psi &= 0, & J^{AB} \Psi &= 0, & \tilde{J}^{A'B'} \Psi &= 0. \end{aligned} \quad (2.5)$$

Starting with the simplest of these, the J^{AB} and $\tilde{J}^{A'B'}$ quantum constraints imply that Ψ is constructed from Lorentz invariants. The $Q_{(a)}$ constraint, derived below, is of first order in functional derivatives, and implies that the wave function Ψ is gauge invariant. The S_A and $\tilde{S}_{A'}$ constraints will be derived and discussed below. The \mathcal{H}_\perp and \mathcal{H}_i constraints can be defined through the anti-commutator of S_A and $\tilde{S}_{A'}$, as in the case of $N=1$ supergravity without matter fields [11]. Thus the remaining constraints imply $\mathcal{H}_\perp \Psi = 0$, $\mathcal{H}_i \Psi = 0$; if one could find a solution of the remaining quantum constraints, the \mathcal{H}_\perp and \mathcal{H}_i constraints would follow (with a certain choice of factor-ordering).

In the Hamiltonian decomposition, the variables are split into the spatial components $e^{AA'}{}_i, \psi^A{}_i, \tilde{\psi}^{A'}{}_i, A_i^{(a)}, \lambda_A^{(a)}, \tilde{\lambda}_{A'}^{(a)}, \Phi^I, \Phi^{J*}, \chi_A^I, \tilde{\chi}_{A'}^{J*}$, which together with the bosonic momenta are the basic dynamical variables of the theory, and the Lagrange multipliers

$N, N^i, \psi^A{}_0, \tilde{\psi}^{A'}{}_0, A_0^{(a)}, M_{AB}, \tilde{M}_{A'B'}$ of Eq. (4), where N, N^i are formed from the $e^{AA'}{}_0$ and the $e^{AA'}{}_i$ [6], and $M_{AB}, \tilde{M}_{A'B'}$ involve the zero components $\omega_{AB0}, \tilde{\omega}_{A'B'0}$ of the connection. One computes the canonical momenta conjugate to the dynamical variables listed above in the usual way. The constraint generators $\mathcal{H}_\perp, \mathcal{H}_i$, etc. are functions of the basic dynamical variables. For the gravitino and spin- $\frac{1}{2}$ fields, the canonical momenta give second-class constraints of the types described in [1,32,33]. These are eliminated when Dirac brackets are introduced [1,32,33] instead of the original Poisson brackets. In particular, one obtains nontrivial Dirac brackets for $p_{AA'}{}^i$, the momentum conjugate to $e^{AA'}{}_i$, for $\psi^A{}_i$ and $\tilde{\psi}^{A'}{}_i$, for $\lambda_A^{(a)}$ and $\tilde{\lambda}_{A'}^{(a)}$, for χ_A^I and $\tilde{\chi}_{A'}^{J*}$, and for π_L, π_{L^*} , the momenta conjugate to Φ^L, Φ^{L^*} . These can be made into simple brackets by three steps.

First, the brackets involving $p_{AA'}{}^i, \psi^A{}_i$ and $\tilde{\psi}^{A'}{}_i$ can be simplified as in the case of pure $N=1$ supergravity [1]. One redefines

$$p_{AA'}{}^i \rightarrow \hat{p}_{AA'}{}^i = p_{AA'}{}^i - \frac{1}{\sqrt{2}} \epsilon^{ijk} \psi_{Aj} \tilde{\psi}_{A'k} . \quad (2.6)$$

This gives the Dirac brackets

$$\begin{aligned} [\hat{p}_{AA'}{}^i, \psi^B{}_j]^* &= 0, & [\hat{p}_{AA'}{}^i, \tilde{\psi}^{B'}{}_j]^* &= 0, \\ [\hat{p}_{AA'}{}^i, \hat{p}_{BB}{}^j]^* &= \text{independent of } \psi^A{}_i \text{ and } \tilde{\psi}^{A'}{}_i. \end{aligned} \quad (2.7)$$

Next, one must deal with a complication caused by the dependence on the scalars Φ^I, Φ^{J*} of the Kähler metric K_{IJ*} in the second-class constraints. Defining π_{IA} to be the momentum conjugate to χ^{IA} , and $\tilde{\pi}_{I^*A'}$ to be the momentum conjugate to $\tilde{\chi}^{I^*A'}$, one has

$$\begin{aligned} \pi_{IA} + \frac{ie}{\sqrt{2}} K_{IJ*} n_{AA'} \tilde{\chi}^{J^*A'} &= 0, \\ \tilde{\pi}_{J^*A'} + \frac{ie}{\sqrt{2}} K_{IJ*} n_{AA'} \chi^{IA} &= 0 \end{aligned} \quad (2.8)$$

where $e = h^{\frac{1}{2}}$, with h the determinant of the spatial metric h_{ij} . Here $n^{AA'}$ is the spinor version of the unit future-directed normal vector n^μ , obeying

$$n_{AA'} n^{AA'} = 1, \quad n_{AA'} e^{AA'}{}_i = 0 . \quad (2.9)$$

The Φ^K and Φ^{K*} dependence of K_{IJ*} is responsible for the unwanted Dirac brackets among $\chi_A^I, \tilde{\chi}_{A'}^{J*}, \pi_L$ and π_{L^*} . One cures this by using the square root of the Kähler metric, $K_{IJ*}^{\frac{1}{2}}$, obeying

$$K_{IJ*}^{\frac{1}{2}} \delta^{KJ*} K_{KL*}^{\frac{1}{2}} = K_{IL*} . \quad (2.10)$$

This may be found by diagonalizing K_{IJ*} via a unitary transformation, assuming that the eigenvalues are all positive. One needs to assume that there is an "identity metric" δ^{KJ*} defined over the Kähler manifold; this will be true if a positive-definite vielbein field can be introduced. One then introduces the modified variables

$$\hat{\chi}_{IA} = e^{\frac{1}{2}} K_{IJ}^{\frac{1}{2}} \delta^{KJ*} \chi_{KA}, \quad \hat{\tilde{\chi}}_{I^*A'} = e^{\frac{1}{2}} K_{J^*I^*}^{\frac{1}{2}} \delta^{JK*} \tilde{\chi}_{K^*A'}, \quad (2.11)$$

where the factor of $e^{1/2}$ has been introduced for later use (in the time gauge). Then the second-class constraints of Eq.(2.8) read

$$\hat{\pi}_{IA} + \frac{i}{\sqrt{2}} \delta_{IJ^*} n_{AA'} \hat{\tilde{\chi}}^{J^*A'} = 0, \quad \hat{\tilde{\pi}}_{I^*A'} + \frac{i}{\sqrt{2}} \delta_{IJ^*} n_{AA'} \hat{\chi}^{JA} = 0. \quad (2.12)$$

The resulting Dirac brackets now give

$$[\pi_L, \pi_M]^* = 0, \quad [\pi_L, \hat{\chi}^A_I]^* = 0, \text{ etc.} \quad (2.13)$$

Finally, there are the brackets among $\hat{p}_{AA'}^i$, $\lambda_A^{(a)}$, $\tilde{\lambda}_{A'}^{(a)}$, $\hat{\chi}_A^I$ and $\hat{\tilde{\chi}}_{A'}^{J^*}$, which are just as in the case studied by Nelson and Teitelboim [32]. These are dealt with by first defining

$$\hat{\lambda}_A^{(a)} = e^{\frac{1}{2}} \lambda_A^{(a)}, \quad \hat{\tilde{\lambda}}_{A'}^{(a)} = e^{\frac{1}{2}} \tilde{\lambda}_{A'}^{(a)}. \quad (2.14)$$

Then one goes to the time gauge, in which the tetrad component n^a of the normal vector n^μ is henceforward restricted by

$$n^a = \delta_0^a, \quad (2.15)$$

or equivalently

$$e^0_i = 0. \quad (2.16)$$

Thus the original Lorentz rotation freedom becomes replaced by that of spatial rotations. In the time gauge, the geometry is described by the triad e^α_i ($\alpha = 1, 2, 3$), and the conjugate momentum is \hat{p}_α^i . One has [33]

$$[\hat{p}_\alpha^i, \hat{p}_\beta^j]^{**} = 0, \quad [\hat{p}_\alpha^i, \hat{\lambda}_A^{(a)}]^{**} = 0, \quad [\hat{p}_\alpha^i, \hat{\chi}_A^I]^{**} = 0, \text{ etc.} \quad (2.17)$$

The remaining brackets are standard; the nonzero fermionic brackets are

$$\left[\hat{\lambda}_A^{(a)}(x), \hat{\tilde{\lambda}}_{A'}^{(b)}(x') \right]^{**} = \sqrt{2} i n_{AA'} \delta^{(a)(b)} \delta(x, x'), \quad (2.18)$$

$$\left[\hat{\chi}_A^I(x), \hat{\tilde{\chi}}_{A'}^{J^*}(x') \right]^{**} = \sqrt{2} i n_{AA'} \delta^{IJ*} \delta(x, x'), \quad (2.19)$$

$$\left[\psi_A^i(x), \tilde{\psi}_{A'}^j(x') \right]^{**} = \frac{1}{\sqrt{2}} D^{AA'}_{ij} \delta(x, x'), \quad (2.20)$$

where

$$D^{AA'}_{ij} = -2i e^{-1} e^{AB'}_j e_{BB'i} n^{BA'}. \quad (2.21)$$

The supersymmetry constraint $\tilde{S}_{A'}$ is then found to be

$$\begin{aligned} \tilde{S}_{A'} = & -\sqrt{2} i \hat{\pi}^{ij} e_{AA'i} \psi^A_j + \sqrt{2} \epsilon^{ijk} e_{AA'i} {}^3\tilde{D}_j \psi^A_k + \left(2\sqrt{2} \right)^{-1} e \tilde{\psi}_{B'}^{[j} \psi_B^{i]} n^{BB'} \psi^A_j e_{AA'i} \\ & - \frac{1}{\sqrt{2}} e \tilde{\psi}_{B'}^{[i} \psi_B^{j]} e^{BB'}_j \psi^A_i n_{AA'} - \frac{i}{\sqrt{2}} \pi^{n(a)} e_{BA'n} \lambda^{(a)B} + \frac{1}{2\sqrt{2}} \epsilon^{ijk} e_{BA'k} \lambda^{(a)B} F_{ij}^{(a)} + \frac{1}{\sqrt{2}} e g D^{(a)} n^A_{A'} \lambda^{(a)}_A \\ & + \frac{1}{\sqrt{2}} \left[\pi_{J^*} - \frac{ie}{2\sqrt{2}} n^{BB'} \lambda^{(a)}_B \tilde{\lambda}^{(a)}_{B'} K_{J^*} + \frac{ie}{\sqrt{2}} g_{LM^*} \Gamma_{J^*N^*}^{M^*} n^{BB'} \tilde{\chi}^{N^*}_{B'} \chi^L_B - \frac{ie}{2\sqrt{2}} K_{J^*} g_{MM^*} n^{BB'} \tilde{\chi}^{M^*}_{B'} \chi^M_B \right. \\ & \left. - \frac{1}{2\sqrt{2}} \epsilon^{ijk} K_{J^*} e^{BB'}_j \psi_{kB} \tilde{\psi}_{iB'} - \sqrt{2} e g_{IJ^*} \chi^{IB} e_{BB'}^m n^{CB'} \psi_{mC} \right] \tilde{\chi}^{J^*}_{A'} \\ & - \sqrt{2} e g_{IJ^*} \left(\tilde{D}_I \Phi^I \right) \tilde{\chi}_{B'}^{J^*} n^{BB'} e_{BA'i} + \frac{i}{2} g_{IJ^*} \epsilon^{ijk} e_{AA'j} \psi^A_i \tilde{\chi}^{J^*B'} e_{BB'k} \chi^{IB} \\ & + \frac{1}{4} e \psi_{Ai} \left(e_{BA'i} n^{AC'} - e^{AC'i} n_{BA'} \right) g_{IJ^*} \tilde{\chi}_{C'}^{J^*} \chi^{IB} + \frac{1}{4} e \psi_{Ai} \left(e_{BA'i} n^{AC'} - e^{AC'i} n_{BA'} \right) \tilde{\chi}_{C'}^{(a)} \lambda^{(a)B} \\ & - e \exp(K/2) \left[2P n^A_{A'} e_{AB'i} \tilde{\psi}^{B'}_i + i(D_IP) n_{AA'} \chi^{IA} \right] \end{aligned} \quad (2.22)$$

where $\lambda_A^{(a)}$, $\tilde{\lambda}_{A'}^{(a)}$ and χ_{IA} , $\tilde{\chi}_{I^*A'}$ should be redefined as in Eqs. (2.11),(2.14). Here $e_{AA'i} = \sigma_{AA'}^\alpha e_{\alpha i}$, where $\sigma_{AA'}^\alpha$ ($\alpha = 1, 2, 3$) are Infeld-van der Waerden symbols [1], and $\hat{\pi}^{ij} = -\frac{1}{2} e^{\alpha(i} \hat{p}_\alpha^{j)}$. Also [27]

$$\begin{aligned}
{}^{3s}\tilde{D}_j\psi^A_k &= \partial_j\psi^A_k + {}^{3s}\omega^A_{Bj}\psi^B_k \\
&+ \frac{1}{4}\left(K_K\tilde{D}_j\Phi^K - K_{K^*}\tilde{D}_j\Phi^{K^*}\right)\psi^A_k \\
&+ \frac{1}{2}gA_j^{(a)}\left(ImF^{(a)}\right)\psi^A_k, \quad (2.23)
\end{aligned}$$

where ${}^{3s}\omega_{ABj}$, ${}^{3s}\tilde{\omega}_{A'B'j}$ give the torsion-free three-dimensional connection, and

$$\tilde{D}_iA^K = \partial_iA^K - gv_i^{(a)}X^{K(a)}, \quad (2.24)$$

with g the gauge coupling constant and $X^{K(a)}$ the a th Killing vector field, as in Eq.(2.2). Further, the analytic functions $F^{(a)}(\Phi^J)$ and $F^{*(a)}(\Phi^{I^*})$ arise [27] from the transformation of the Kähler potential K under an isometry generated by the Killing vectors $X^{(a)}$ and $X^{*(a)}$:

$$\delta K = \left(\epsilon^{(a)}X^{(a)} + \epsilon^{*(a)}X^{*(a)}\right)K. \quad (2.25)$$

One obtains

$$\delta K = \epsilon^{(a)}F^{(a)} - \epsilon^{*(a)}F^{*(a)} - i\left(\epsilon^{(a)} - \epsilon^{*(a)}\right)D^{(a)}, \quad (2.26)$$

where $D^{(a)}$ is the Killing potential of Eq.(2.3). Also, in Eq.(2.22), $\pi^{n(a)}$ is the momentum conjugate to $A_n^{(a)}$, K_{J^*} denotes $\partial K/\partial\Phi^{J^*}$, $\Gamma_{J^*N^*}^{M^*}$ denotes the starred Christoffel symbols [27] of the Kähler geometry, and $P = P(\Phi^I)$ gives the potential of the theory.

The gauge generator $Q^{(a)}$ is given classically by

$$\begin{aligned}
Q^{(a)} &= -\partial_n\pi^{n(a)} - g f^{abc}\pi^{n(b)}A_n^{(c)} \\
&+ g\left(\pi_I X^{I(a)} + \pi_{I^*} X^{I^*(a)}\right) \\
&+ \sqrt{2}iegK_{MI^*}n^{AA'}X^{J^*(a)}\Gamma_{J^*N^*}^{I^*}\tilde{\chi}_{A'}^{N^*}\chi_A^M \\
&- \sqrt{2}iegn^{AA'}\tilde{\lambda}_{A'}^{(b)}\left[f^{abc}\lambda_A^{(c)} + \frac{i}{2}\left(ImF^{(a)}\right)\lambda_A^{(b)}\right] \\
&+ \sqrt{2}iegn^{AA'}K_{IJ^*}\tilde{\chi}_{A'}^{J^*}\left[\frac{\partial X^{I(a)}}{\partial A^J}\chi_A^J + \frac{i}{2}\left(ImF^{(a)}\right)\chi_A^I\right] \\
&- \frac{i}{\sqrt{2}}g\left(ImF^{(a)}\right)\epsilon^{ijk}\tilde{\psi}_{iA'}e^{AA'}_j\psi_{Ak}, \quad (2.27)
\end{aligned}$$

where f^{abc} are the structure constants of the isometry group.

One can proceed to a quantum description by studying (for example) Grassmann-algebra-valued wave functions of the form $\Psi(e^\alpha_i, \psi^A_i, A^{(a)}_i, \hat{\lambda}^{(a)}_i, n_{A'}^{A'}\tilde{\chi}_{A'}^{J^*}, \Phi^I, \Phi^{J^*})$. The choice of $n_{A'}^{A'}\tilde{\chi}_{A'}^{J^*}$ rather than $\hat{\chi}_{A'}^J$ is designed so that the quantum constraint $\bar{S}_{A'}$ should be of first order in momenta. The momenta are represented by

$$\hat{p}_\alpha^i \rightarrow -i\hbar\delta/\delta e^\alpha_i, \quad \pi^{n(a)} \rightarrow -i\hbar\delta/\delta A_n^{(a)}, \quad (2.28a)$$

$$\pi_I \rightarrow -i\hbar\delta/\delta\Phi^I, \quad \pi_{I^*} \rightarrow -i\hbar\delta/\delta\Phi^{I^*}, \quad (2.28b)$$

$$\bar{\psi}^{A'}_i \rightarrow \frac{1}{\sqrt{2}}i\hbar D^{AA'}_{ji}\delta/\delta\psi^A_j, \quad (2.29a)$$

$$\frac{\hat{\lambda}^{(a)A'}}{\lambda} \rightarrow -\sqrt{2}n^{AA'}\delta/\delta\hat{\lambda}^{(a)}, \quad (2.29b)$$

$$\hat{\chi}^{IA} \rightarrow -\sqrt{2}n^{AA'}\delta^{IJ^*}\delta/\delta\hat{\chi}^{J^*A'}. \quad (2.29c)$$

Quantum-mechanically, we order each term cubic in fermions in $\bar{S}_{A'}$ (using anti-commutation) such that one ‘‘momentum’’ fermionic variable is on the right, and two ‘‘coordinate’’ fermionic variables are on the left. The ordering of the quantum constraint S_A is defined by taking the hermitian adjoint with respect to the natural inner product [1,35]. Then the terms in S_A cubic in fermions have two ‘‘momenta’’ on the right and one ‘‘coordinate’’ on the left.

III. Ansätze for the Fields and Dimensional Reduction

In order to learn more about quantum cosmology with local supersymmetry we would like to study certain types of simple mini-superspace models. Among the simplest non-trivial mini-superspace models (in which the gravitational and matter variables have been reduced to a finite number of degrees of freedom) are those based on Friedmann universes with S^3 spatial sections, which are the spatial orbits of $G = SO(4)$ – the group of homogeneity and isotropy. Consistent with this assumption we choose the geometry to be that of a $k = +1$ Friedmann model. The tetrad of the four-dimensional theory is taken to be:

$$\begin{aligned}
e_{a\mu} &= \begin{pmatrix} N(\tau) & 0 \\ N^i a(\tau)E_{\hat{a}i} & a(\tau)E_{\hat{a}i} \end{pmatrix}, \\
e^{a\mu} &= \begin{pmatrix} N(\tau)^{-1} & -N^i N(\tau)^{-1} \\ 0 & a(\tau)^{-1}E^{\hat{a}i} \end{pmatrix} \quad (3.1)
\end{aligned}$$

where \hat{a} and i run from 1 to 3. The shift vector N^i is assumed to take the form $N^i = -N^{AA'}(\tau)e_{AA'}^i$, where $N^{AA'}n_{AA'} = 0$.

$E_{\hat{a}i}$ is a basis of left-invariant 1-forms on the unit S^3 with volume $\sigma^2 = 2\pi^2$. The spatial tetrad $e^{AA'}_i$ satisfies the relation

$$\partial_i e^{AA'}_j - \partial_j e^{AA'}_i = 2a^2 e_{ijk} e^{AA'k} \quad (3.2)$$

as a consequence of the group structure of $SO(3)$, the isotropy (sub)group.

This Ansatz reduces the number of degrees of freedom provided by $e_{AA'\mu}$. If supersymmetry invariance is to be retained, then we need an Ansatz for ψ^A_μ and $\tilde{\psi}^{A'}_\mu$ which reduces the number of fermionic degrees of freedom, so that there is equality between the number of bosonic and fermionic degrees of freedom. One is naturally led to take ψ^A_0 and $\tilde{\psi}^{A'}_0$ to be functions of

time only. In the four-dimensional Hamiltonian theory, ψ^A_0 and $\tilde{\psi}^{A'}_0$ are Lagrange multipliers which may be freely specified. For this reason we do not allow ψ^A_0 and $\tilde{\psi}^{A'}_0$ to depend on ψ^A_i or $\tilde{\psi}^{A'}_i$ in our Ansatz. We further take

$$\psi^A_i = e^{AA'}_i \tilde{\psi}^{A'}, \quad \tilde{\psi}^{A'}_i = e^{AA'}_i \psi_A, \quad (3.3)$$

where we introduce the new spinors ψ_A and $\tilde{\psi}_{A'}$ which are functions of time only. [It is possible to justify the Ansatz (3.3) by requiring that the form (3.1) of the tetrad be preserved under suitable homogeneous supersymmetry transformations [4,5,7].] Moreover, it turns out that the constraints obeyed by classical solutions of the 1-dimensional theory lead to a 4-dimensional energy-momentum tensor which is isotropic, consistent with the assumption of a Friedmann geometry.

It is important to remark that the Ansatz for ψ^A_i is preserved under a combination of a non-zero (spatially homogeneous) supersymmetry transformation and possible local Lorentz and coordinate transformations [4,5,7] if we impose the additional constraint

$$\psi^B \tilde{\psi}^{B'} e_{BB'i} = 0. \quad (3.4)$$

Eq. (3.4) implies that $\psi^B \tilde{\psi}^{B'} \propto n^{BB'}$, and so we can write (2.8) in the two equivalent forms:

$$J_{AB} = \psi_{(A} \tilde{\psi}^{B'} n_{B)B'} = 0, \quad (3.5)$$

$$\tilde{J}_{A'B'} = \tilde{\psi}_{(A'} \psi^B n_{B)B'} = 0. \quad (3.6)$$

The constraint $J_{AB} = 0$ has a natural interpretation as the reduced form of the Lorentz rotation constraint arising in the full theory [1]. By requiring that the constraint $J_{AB} = 0$ be preserved under the same combination of transformations as used above, one finds equations which are satisfied provided the supersymmetry constraints $S_A = 0$, $\tilde{S}_{A'} = 0$ (see below) hold. By further requiring that the supersymmetry constraints be preserved, one finds additionally that the Hamiltonian constraint $\mathcal{H} = 0$ should hold. When matter fields are taken into account (see next paragraphs) the generalisation of the J_{AB} constraint is :

$$J_{AB} = \psi_{(A} \tilde{\psi}_{B)} - \chi_{(A} \tilde{\chi}_{B)} - \lambda_{(A}^{(a)} \tilde{\lambda}_{B)}^{(a)} = 0. \quad (3.7)$$

One can justify this by observing either that it arises from the corresponding constraint of the full theory, or that its quantum version describes the invariance of the wavefunction under Lorentz transformations.

Now, let us address the supermatter fields. First, we choose for the gauge group of our model the group $\hat{G} = SU(2)$. In this case we have that [27]

$$D^{(1)} = \frac{1}{2} \left(\frac{\phi + \bar{\phi}}{1 + \phi\bar{\phi}} \right), \quad D^{(2)} = -\frac{i}{2} \left(\frac{\phi - \bar{\phi}}{1 + \phi\bar{\phi}} \right), \\ D^{(3)} = -\frac{1}{2} \left(\frac{1 - \phi\bar{\phi}}{1 + \phi\bar{\phi}} \right), \quad (3.8)$$

and

$$K = \ln(1 + \phi\bar{\phi}), \quad (3.9)$$

and thus

$$g_{\phi\bar{\phi}} = \frac{1}{(1 + \phi\bar{\phi})^2}, \quad g^{\phi\bar{\phi}} = (1 + \phi\bar{\phi})^2. \quad (3.10)$$

The Levi-Civita connections of the S^2 Kähler manifold are

$$\Gamma_{\phi\phi}^{\phi} = g^{\phi\bar{\phi}} \frac{\partial g_{\phi\bar{\phi}}}{\partial \phi} = -2 \frac{\bar{\phi}}{(1 + \phi\bar{\phi})} \quad (3.11)$$

and its complex conjugate. The rest of the components are zero. The scalar super-multiplet, consisting of a complex massive scalar field ϕ and massive spin-1/2 field $\chi, \tilde{\chi}$ are chosen to be spatially homogeneous, depending only on time. The odd spin- $\frac{1}{2}$ partner $(\lambda^{(a)}, \tilde{\lambda}^{(a)})$, $a = 1, 2, 3$, are chosen to depend only on time as well. As far as the vector field $A_{\mu}^{(a)}$ is concerned we adopt here the ansatz formulated in Refs. [29,30,31]. More specifically, since the physical observables to be $SO(4)$ -invariant, the fields with gauge degrees of freedom may transform under $SO(4)$ if these transformations can be compensated by a gauge transformation. This is so since the physical observables are gauge invariant quantities. Fortunately there is a large class of fields satisfying the above conditions. These are the so-called $SO(4)$ -symmetric fields, i.e. fields which are invariant up to a gauge transformation. According to group theory considerations [29,30,31] the $SO(4)$ -symmetric spin-1 field is taken to be

$$\mathbf{A}_{\mu}(t) \omega^{\mu} = \left(\frac{f(t)}{4} \varepsilon_{acb} \mathcal{T}_{ab} \right) \omega^c \quad (3.12)$$

where $\{\omega^{\mu}\}$ represents the moving coframe $\{\omega^{\mu}\} = \{dt, \omega^b\}$, ($b = 1, 2, 3$), of one-forms, invariant under the left action of $SU(2)$ and \mathcal{T}_{ab} are the generators of the $SU(2)$ gauge group. The idea behind this Ansatz for a non-Abelian spin-1 field is to define a homomorphism of the isotropy group $SO(3)$ to the gauge group. This homomorphism defines the gauge transformation which, for the symmetric fields, compensates the action of a given $SO(3)$ rotation. Hence, the above form for the gauge field where the A_0 component is taken to be identically zero.

If one assumes that the dynamics of the most general $N = 1$ supergravity theory coupled to supermatter is as given in Eq.(25.12) of [27] than, by imposing the above mentioned symmetry conditions, we obtain a one-dimensional (mechanical) model depending only on t . The resulting one-dimensional model will have some symmetries remaining from the symmetries of the four-dimensional theory. In particular the invariance under general coordinate transformations in four dimensions leads to an invariance under arbitrary time-reparametrizations. However, due

to our choice of $SO(4)$ -symmetry conditions on the spin-1 field, none of the local internal (i.e. gauge) symmetries will survive: all the available gauge transformations are required to cancel out the action of a given $SO(3)$ rotation. Thus, we will not have in our FRW case a gauge constraint $Q^{(a)} = 0$ [29–31]. However, in the case of larger gauge group some of the gauge symmetries will survive, giving rise, in the one-dimensional model, to local internal symmetries with a reduced gauge group. Therefore, a gauge constraint can be expected to play an important role in such a case [13,29–31] and a study of such a model would be interesting.

In the next section we will study associated FRW cosmological model. From the one-dimensional effective action we will derive the supersymmetry constraints of our theory.

IV. Supersymmetry Constraints in the One-Dimensional Theory

Using the Ansätze described in the previous section, the action of the full theory (Eq.(25.12) in Ref.[27]) is reduced to one with a finite number of degrees of freedom. Starting from the action so obtained, we study the Hamiltonian formulation of this model. As discussed above, the Hamiltonian of any supersymmetric model has the form (2.4). The procedure to find the expressions of S_A and $\bar{S}_{A'}$ is very simple. First, we have to calculate the conjugate momenta of the dynamical variables and then evaluate the usual expression:

$$H(p_i, q^i) = p_i \dot{q}^i - L. \quad (4.1)$$

Afterwards, we read out the coefficients of ψ_0^A and $\bar{\psi}_0^{A'}$ from this expression in order to get the S_A and $\bar{S}_{A'}$ constraints, respectively.

The contributions from the spin-0 field ϕ to the $\bar{S}_{A'}$ constraint are seen to be

$$\begin{aligned} & \frac{1}{\sqrt{2}} \bar{\chi}_{A'} \left[\pi_{\bar{\phi}} + \frac{i\sigma^2 a^3}{2\sqrt{2}} \frac{\phi}{(1+\phi\bar{\phi})} n_{BB'} \bar{\lambda}^{(a)B'} \lambda^{(a)B} \right. \\ & \quad \left. - \frac{5i}{2\sqrt{2}} \frac{\sigma^2 a^3 \phi}{(1+\phi\bar{\phi})^3} n_{BB'} \bar{\chi}^{B'} \chi^B \right. \\ & \quad \left. - \frac{3i}{2\sqrt{2}} \frac{\sigma^2 a^3 \phi}{(1+\phi\bar{\phi})} n_{BB'} \psi^B \bar{\psi}^{B'} - \frac{3}{\sqrt{2}} \frac{\sigma^2 a^3}{(1+\phi\bar{\phi})^2} n_{BB'} \chi^B \bar{\psi}^{B'} \right] \\ & \quad + \frac{\sigma^2 a^2 g f}{\sqrt{2}(1+\phi\bar{\phi})^2} \sigma^a_{AA'} n^{AB'} \bar{\chi}_{B'} X^a. \end{aligned} \quad (4.2)$$

The contributions to the $\bar{S}_{A'}$ constraint from the spin-1 field are

$$\begin{aligned} & -i \frac{\sqrt{2}}{3} \pi_f \sigma^a_{BA'} \lambda^{(a)B} + \frac{\sigma^2 a^3}{6} \sigma^a_{BA'} \lambda^{(a)B} n_{CB'} \\ & \quad \times (\sigma^{bCC'} \bar{\psi}_{C'} \bar{\lambda}^{(b)B'} + \sigma^{bAB'} \psi_A \lambda^{(b)C}) \\ & \quad + \frac{1}{8\sqrt{2}} \sigma^2 a^4 \sigma^a_{A'} [1 - (f-1)^2] \lambda^{(a)}_C \end{aligned}$$

$$\begin{aligned} & + \frac{1}{2} \sigma^2 a^3 \lambda^{(a)A} \left(-n_{AB'} \bar{\psi}_{A'} \bar{\lambda}^{(a)B'} + n_{BA'} \psi_A \lambda^{(a)B} \right. \\ & \quad \left. - \frac{1}{2} n_{AA'} \psi_B \lambda^{(a)B} + \frac{1}{2} n_{AA'} \bar{\psi}_{B'} \bar{\lambda}^{(a)B'} \right). \end{aligned} \quad (4.3)$$

The contributions from the spin-2 field and spin-3/2 field to $\bar{S}_{A'}$ constraint are

$$\frac{i}{2\sqrt{2}} a \pi_a \bar{\psi}_{A'} - \frac{3}{\sqrt{2}} \sigma^2 a^2 \bar{\psi}_{A'} + \frac{3}{8} \sigma^2 a^3 n^B_{A'} \bar{\psi}^{B'} \psi_B \bar{\psi}_{B'}. \quad (4.4)$$

The following terms are also present in the $\bar{S}_{A'}$ supersymmetry constraint:

$$\begin{aligned} & - \frac{1}{\sqrt{2}} \sigma^2 a^3 g D^a n_{AA'} \lambda^{(a)A} \\ & \quad + \frac{\sigma^2 a^3}{(1+\phi\bar{\phi})^2} (-n_{BA'} \bar{\psi}_{B'} + \frac{1}{2} n_{BB'} \bar{\psi}_{A'}) \chi^B \bar{\chi}^{B'} \\ & \quad - \frac{1}{4} \sigma^2 a^3 (n_{AB'} \lambda^{(a)A} \bar{\lambda}^{(a)}_{A'} \bar{\psi}^{B'} + n_{AA'} \lambda^{(a)A} \bar{\lambda}^{(a)}_{B'} \bar{\psi}^{B'}) \\ & \quad - \frac{1}{4(1+\phi\bar{\phi})^2} \sigma^2 a^3 (n_{AB'} \chi^A \bar{\chi}_{A'} \psi^{B'} + n_{AA'} \chi^A \bar{\chi}_{B'} \psi^{B'}). \end{aligned} \quad (4.5)$$

The supersymmetry constraint $\bar{S}_{A'}$ is then the sum of the above expressions. The supersymmetry constraint S_A is just the complex conjugate of $\bar{S}_{A'}$.

V. Solutions of the Supersymmetry Constraints

In this section we will solve explicitly the corresponding quantum supersymmetry constraints but we will rather restrict ourselves to the case where both the spin-1 field and its fermionic partner equal to zero. For the cases of spherically symmetric and flat Kähler geometries we have found the general solution for the quantum state with a very simple form. However, these states are *somewhat different* from the ones presented in Refs. [4–7]; although they allow a Hartle-Hawking solution, we cannot find a wormhole state.

First we need to redefine the χ_A field and ψ_A field in order to simplify the Dirac brackets [13,23], following some of the steps described in section II:

$$\hat{\chi}_A = \frac{\sigma a^{\frac{3}{2}}}{2^{\frac{1}{4}}(1+\phi\bar{\phi})} \chi_A, \quad \hat{\chi}_{A'} = \frac{\sigma a^{\frac{3}{2}}}{2^{\frac{1}{4}}(1+\phi\bar{\phi})} \bar{\chi}_{A'}. \quad (5.1)$$

The conjugate momenta become

$$\pi_{\hat{\chi}_A} = -i n_{AA'} \hat{\chi}^{A'}, \quad \pi_{\hat{\chi}_{A'}} = -i n_{AA'} \hat{\chi}^A. \quad (5.2)$$

This pair form a set of second class constraints. Consequently, the Dirac bracket (DB) becomes

$$[\hat{\chi}_A, \hat{\chi}_{A'}]_{DB} = -i n_{AA'}. \quad (5.3)$$

Similarly for the ψ_A field,

$$\hat{\psi}_A = \frac{\sqrt{3}}{2^{\frac{1}{4}}} \sigma a^{\frac{3}{2}} \psi_A, \quad \hat{\psi}_{A'} = \frac{\sqrt{3}}{2^{\frac{1}{4}}} \sigma a^{\frac{3}{2}} \bar{\psi}_{A'} \quad (5.4)$$

where the conjugate momenta are

$$\pi_{\hat{\psi}_A} = in_{AA'}\hat{\psi}^{A'}, \quad \pi_{\hat{\psi}_{A'}} = in_{AA'}\hat{\psi}^A. \quad (5.5)$$

The Dirac bracket is then

$$[\hat{\psi}_A, \hat{\psi}_{A'}]_{DB} = in_{AA'}. \quad (5.6)$$

Furthermore,

$$[a, \pi_a]_{DB} = 1, \quad [\phi, \pi_\phi]_{DB} = 1, \quad [\bar{\phi}, \pi_{\bar{\phi}}]_{DB} = 1 \quad (5.7)$$

and the rest of the brackets are zero.

After substituting the redefined fields in the constraints, we drop the hat over the new variables. The supersymmetry constraints become

$$\begin{aligned} \bar{S}_{A'} &= \frac{1}{\sqrt{2}}(1 + \phi\bar{\phi})\bar{\chi}_{A'}\pi_{\bar{\phi}} + \frac{i}{2\sqrt{6}}a\pi_a\bar{\psi}_{A'} \\ &- \sqrt{\frac{3}{2}}\sigma^2 a^2\bar{\psi}_{A'} - \frac{5i}{2\sqrt{2}}\phi n_{BB'}\bar{\chi}^{B'}\chi^B\bar{\chi}_{A'} \\ &- \frac{1}{4\sqrt{6}}n_{BA'}\bar{\psi}^{B'}\psi^B - \frac{i}{2\sqrt{2}}\phi n_{BB'}\psi^B\bar{\psi}^{B'}\bar{\chi}_{A'} \\ &- \frac{5}{2\sqrt{6}}n_{BB'}\chi^B\bar{\psi}^{B'}\bar{\chi}_{A'} - \frac{\sqrt{3}}{2\sqrt{2}}n_{AA'}\bar{\psi}_{B'}\chi_A\bar{\chi}^{B'} \\ &+ \frac{1}{\sqrt{6}}n_{BB'}\chi^B\bar{\chi}^{B'}\bar{\psi}_{A'} \end{aligned} \quad (5.8)$$

together with its complex conjugate.

It is simpler to describe the theory using only (say) unprimed spinors, and, to this end, we define

$$\bar{\psi}_A = 2n_A^{B'}\bar{\psi}_{B'}, \quad \bar{\chi}_A = 2n_A^{B'}\bar{\chi}_{B'} \quad (5.9)$$

with which the new Dirac brackets are

$$[\chi_A, \bar{\chi}_B]_{DB} = -i\epsilon_{AB}, \quad [\psi_A, \bar{\psi}_B]_{DB} = -i\epsilon_{AB}. \quad (5.10)$$

The rest of the brackets remain unchanged. Using these new variables, the supersymmetry constraints are

$$\begin{aligned} S_A &= \frac{1}{\sqrt{2}}(1 + \phi\bar{\phi})\chi_A\pi_\phi - \frac{i}{2\sqrt{6}}a\pi_a\psi_A - \sqrt{\frac{3}{2}}\sigma^2 a^2\psi_A \\ &- \frac{5i}{4\sqrt{2}}\bar{\phi}\chi_A\bar{\chi}_B\chi^B + \frac{1}{8\sqrt{6}}\psi_B\bar{\psi}_A\psi^B \\ &- \frac{i}{4\sqrt{2}}\bar{\phi}\chi_A\psi^B\bar{\psi}_B + \frac{5}{4\sqrt{6}}\chi_A\psi^B\bar{\chi}_B \\ &+ \frac{\sqrt{3}}{4\sqrt{2}}\chi^B\bar{\chi}_A\psi_B - \frac{1}{2\sqrt{6}}\psi_A\chi^B\bar{\chi}_B \end{aligned} \quad (5.11)$$

and

$$\begin{aligned} \bar{S}_A &= \frac{1}{\sqrt{2}}(1 + \phi\bar{\phi})\bar{\chi}_A\pi_{\bar{\phi}} + \frac{i}{2\sqrt{6}}a\pi_a\bar{\psi}_A - \sqrt{\frac{3}{2}}\sigma^2 a^2\bar{\psi}_A \\ &+ \frac{5i}{4\sqrt{2}}\phi\bar{\chi}_B\chi^B\bar{\chi}_A - \frac{1}{8\sqrt{6}}\bar{\psi}^B\psi_A\bar{\psi}_B \\ &- \frac{i}{4\sqrt{2}}\phi\psi_B\bar{\psi}^B\bar{\chi}_A + \frac{5}{4\sqrt{6}}\chi^B\bar{\psi}_B\bar{\chi}_A \\ &- \frac{\sqrt{3}}{4\sqrt{2}}\bar{\psi}_B\chi_A\bar{\chi}^B - \frac{1}{2\sqrt{6}}\chi^B\bar{\chi}_B\bar{\psi}_A \end{aligned} \quad (5.12)$$

Quantum mechanically, one replaces the Dirac brackets by anti-commutators if both arguments are odd (O) or commutators if otherwise (E):

$$\begin{aligned} [E_1, E_2] &= i[E_1, E_2]_{DB}, \quad [O, E] = i[O, E]_{DB}, \\ \{O_1, O_2\} &= i[O_1, O_2]_{DB}. \end{aligned} \quad (5.13)$$

Here, we use the convention $\hbar = 1$. And the only non-zero (anti-)commutators relations are:

$$\begin{aligned} \{\chi_A, \bar{\chi}_A\} &= \epsilon_{AB}, \quad \{\psi_A, \bar{\psi}_A\} = -\epsilon_{AB}, \\ [a, \pi_a] &= i, \quad [\phi, \pi_\phi] = i, \quad [\bar{\phi}, \pi_{\bar{\phi}}] = i. \end{aligned} \quad (5.14)$$

Here we choose $(\chi_A, \psi_A, a, \phi, \bar{\phi})$ to be the coordinates of the configuration space, and $(\bar{\chi}_A, \bar{\psi}_A, \pi_a, \pi_\phi, \pi_{\bar{\phi}})$ to be the momentum operators in this representation. Hence

$$\begin{aligned} \bar{\chi}_A &\rightarrow \frac{\partial}{\partial\chi_A}, \quad \bar{\psi}_A \rightarrow \frac{\partial}{\partial\psi_A}, \\ \pi_a &\rightarrow \frac{\partial}{\partial a}, \quad \pi_\phi \rightarrow -i\frac{\partial}{\partial\phi}, \quad \pi_{\bar{\phi}} \rightarrow -i\frac{\partial}{\partial\bar{\phi}} \end{aligned} \quad (5.15)$$

Following the ordering used in Ref.[5], we put all the fermionic derivatives in S_A on the right. In \bar{S}_A , all the fermionic derivatives are on the left. And the supersymmetry generators have the operator form

$$\begin{aligned} S_A &= -\frac{i}{\sqrt{2}}(1 + \phi\bar{\phi})\chi_A\frac{\partial}{\partial\phi} - \frac{1}{2\sqrt{6}}a\psi_A\frac{\partial}{\partial a} \\ &- \sqrt{\frac{3}{2}}\sigma^2 a^2\psi_A - \frac{5i}{4\sqrt{2}}\bar{\phi}\chi_A\chi^B\frac{\partial}{\partial\chi^B} \\ &- \frac{1}{8\sqrt{6}}\psi_B\psi^B\frac{\partial}{\partial\psi^A} - \frac{i}{4\sqrt{2}}\bar{\phi}\chi_A\psi^B\frac{\partial}{\partial\psi^B} \\ &- \frac{5}{4\sqrt{6}}\chi_A\psi^B\frac{\partial}{\partial\chi^B} + \frac{\sqrt{3}}{4\sqrt{2}}\chi^B\psi_B\frac{\partial}{\partial\chi^A} + \frac{1}{2\sqrt{6}}\psi_A\chi^B\frac{\partial}{\partial\chi^B} \end{aligned} \quad (5.16a)$$

and

$$\begin{aligned} \bar{S}_A &= \frac{i}{\sqrt{2}}(1 + \phi\bar{\phi})\frac{\partial}{\partial\chi^A}\frac{\partial}{\partial\bar{\phi}} + \frac{1}{2\sqrt{6}}a\frac{\partial}{\partial a}\frac{\partial}{\partial\bar{\psi}^A} \\ &- \sqrt{\frac{3}{2}}\sigma^2 a^2\frac{\partial}{\partial\bar{\psi}^A} + \frac{5i}{4\sqrt{2}}\phi\frac{\partial}{\partial\chi^A}\frac{\partial}{\partial\chi^B}\chi^B \\ &- \frac{1}{8\sqrt{6}}\epsilon^{BC}\frac{\partial}{\partial\bar{\psi}^B}\frac{\partial}{\partial\chi^C}\psi_A - \frac{i}{4\sqrt{2}}\phi\frac{\partial}{\partial\bar{\psi}^B}\frac{\partial}{\partial\chi^A}\psi^B \\ &- \frac{5}{4\sqrt{6}}\frac{\partial}{\partial\bar{\psi}^B}\frac{\partial}{\partial\chi^A}\chi^B - \frac{\sqrt{3}}{4\sqrt{2}}\epsilon^{BC}\frac{\partial}{\partial\bar{\psi}^B}\frac{\partial}{\partial\chi^C}\chi_A \\ &\frac{1}{2\sqrt{6}}\frac{\partial}{\partial\bar{\psi}^A}\frac{\partial}{\partial\chi^B}\chi^B \end{aligned} \quad (5.16b)$$

We now proceed to find the wavefunction of our model. The Lorentz constraint J_{AB} is easy to solve. It tells us the wave function should be a Lorentz scalar. In our model, $J_{AB} = \psi_{(A}\bar{\psi}_{B)} - \chi_{(A}\bar{\chi}_{B)}$. We can easily see that the most general form of the wave function which satisfies the Lorentz constraint is

$$\begin{aligned} \Psi &= A + iB\psi^C\psi_C + C\psi^C\chi_C \\ &+ iD\chi^C\chi_C + E\psi^C\psi_C\chi^C\chi_C \end{aligned} \quad (5.17)$$

where A, B, C, D , and E are functions of a, ϕ and $\bar{\phi}$ only. The factors of i are chosen to simplify the future results. The next step is to solve the supersymmetry constraints $S_A\Psi = 0$ and $\bar{S}_{A'}\Psi = 0$. Since the wave function (5.17) is of even order in fermionic variables and stops at order four, the $S_A\Psi = 0$ and $\bar{S}_{A'}\Psi = 0$ will be of odd order in fermionic variables and stop at order three. Hence we will get four equations from $S_A\Psi = 0$ and other four equations from $\bar{S}_{A'}\Psi = 0$.

$$-\frac{i}{\sqrt{2}}(1 + \phi\bar{\phi})\frac{\partial A}{\partial\phi} = 0, \quad (5.18a)$$

$$-\frac{a}{2\sqrt{6}}\frac{\partial A}{\partial a} - \sqrt{\frac{3}{2}}\sigma^2 a^2 A = 0, \quad (5.18b)$$

$$(1 + \phi\bar{\phi})\frac{\partial B}{\partial\phi} + \frac{1}{2}\bar{\phi}B + \frac{a}{4\sqrt{3}}\frac{\partial C}{\partial a} - \frac{7}{4\sqrt{3}}C + \frac{\sqrt{3}}{2}\sigma^2 a^2 C = 0, \quad (5.18c)$$

$$\frac{a}{\sqrt{3}}\frac{\partial D}{\partial a} + 2\sqrt{3}\sigma^2 a^2 D - \sqrt{3}D - (1 + \phi\bar{\phi})\frac{C}{\phi} - \frac{3}{2}\bar{\phi}C = 0, \quad (5.18d)$$

$$i\sqrt{2}(1 + \phi\bar{\phi})\frac{\partial E}{\partial\phi} = 0, \quad (5.19a)$$

$$\frac{a}{\sqrt{6}}\frac{\partial E}{\partial a} - \sqrt{6}\sigma^2 a^2 E = 0, \quad (5.19b)$$

$$\frac{a}{\sqrt{3}}\frac{\partial B}{\partial a} - 2\sqrt{3}\sigma^2 a^2 B - \sqrt{3}B + (1 + \phi\bar{\phi})\frac{\partial C}{\partial\phi} + \frac{3}{2}\phi C = 0, \quad (5.19c)$$

$$(1 + \phi\bar{\phi})\frac{\partial D}{\partial\phi} + \frac{1}{2}\phi D - \frac{a}{4\sqrt{3}}\frac{\partial C}{\partial a} + \frac{7}{4\sqrt{3}}C + \frac{\sqrt{3}}{2}\sigma^2 a^2 C = 0. \quad (5.19d)$$

We can see that (5.18a), (5.18b) and (5.19a), (5.19b) constitute decoupled equations for A and E , respectively. They have the general solution.

$$A = f(\bar{\phi})\exp(-3\sigma^2 a^2), \quad E = g(\phi)\exp(3\sigma^2 a^2) \quad (5.20)$$

where f, g are arbitrary anti-holomorphic and holomorphic functions of ϕ , respectively. Eqs.(5.18c) and (5.18d) are coupled equations between B and C and Eqs.(5.19c) and (5.19d) are coupled equations between C and D . The first step to decouple these equations is as follows. Let $B = \tilde{B}(1 + \phi\bar{\phi})^{-\frac{1}{2}}$, $C = \frac{\tilde{C}}{\sqrt{3}}(1 + \phi\bar{\phi})^{-\frac{3}{2}}$, $D = \tilde{D}(1 + \phi\bar{\phi})^{-\frac{1}{2}}$. Equations (5.18c), (5.18d), (5.19c) and (5.19d) then become

$$(1 + \phi\bar{\phi})^2\frac{\partial\tilde{B}}{\partial\phi} + \frac{a}{12}\frac{\partial\tilde{C}}{\partial a} - \frac{7}{12}\tilde{C} + \frac{1}{2}\sigma^2 a^2\tilde{C} = 0, \quad (5.21a)$$

$$(1 + \phi\bar{\phi})^2\frac{\partial\tilde{D}}{\partial\phi} - \frac{a}{12}\frac{\partial\tilde{C}}{\partial a} + \frac{7}{12}\tilde{C} + \frac{1}{2}\sigma^2 a^2\tilde{C} = 0, \quad (5.21b)$$

$$\frac{\partial\tilde{C}}{\partial\phi} - a\frac{\partial\tilde{D}}{\partial a} - 6\sigma^2 a^2\tilde{D} + 3\tilde{D} = 0, \quad (5.21c)$$

$$\frac{\partial\tilde{C}}{\partial\phi} + a\frac{\partial\tilde{B}}{\partial a} - 6\sigma^2 a^2\tilde{B} - 3\tilde{B} = 0. \quad (5.21d)$$

From (5.21a) and (5.21d), we can eliminate \tilde{B} to get a partial differential equation for \tilde{C} :

$$(1 + \phi\bar{\phi})^2\frac{\partial\tilde{C}}{\partial\bar{\phi}\partial\phi} - \frac{a}{12}\frac{\partial}{\partial a}\left(a\frac{\partial\tilde{C}}{\partial a}\right) + \frac{5}{6}a\frac{\partial\tilde{C}}{\partial a} + \left[3\sigma^4 a^4 + 3\sigma^2 a^2 - \frac{7}{4}\right]\tilde{C} = 0 \quad (5.22)$$

and from (5.21b) and (5.21c), we will get another partial differential equation for \tilde{C} :

$$(1 + \phi\bar{\phi})^2\frac{\partial\tilde{C}}{\partial\bar{\phi}\partial\phi} - \frac{a}{12}\frac{\partial}{\partial a}\left(a\frac{\partial\tilde{C}}{\partial a}\right) + \frac{5}{6}a\frac{\partial\tilde{C}}{\partial a} + \left[3\sigma^4 a^4 - 3\sigma^2 a^2 - \frac{7}{4}\right]\tilde{C} = 0. \quad (5.23)$$

We can see immediately that $\tilde{C} = 0$ because the coefficients of $\sigma^2 a^2 \tilde{C}$ are different for these two equations. Using this result, we find

$$B = h(\bar{\phi})(1 + \phi\bar{\phi})^{-\frac{1}{2}}a^3 \exp(3\sigma^2 a^2),$$

$$C = 0, \quad D = k(\phi)(1 + \phi\bar{\phi})^{-\frac{1}{2}}a^3 \exp(-3\sigma^2 a^2). \quad (5.24)$$

This results can be strengthened as we will show that $\tilde{C} = 0$ is not a result of the particular ordering used in the above calculations. In fact, we can try the ordering presented in Ref. [6] such that S_A and $\bar{S}_{A'}$ are hermitian adjoints in the standard inner product, appropriate to the holomorphic representation being used here for the fermions. If one allows for the factor ordering ambiguity in S_A due to the terms cubic in fermions, and insists that $\bar{S}_{A'}$ be the hermitian adjoint of S_A , the operators have the form

$$S_{A(\text{new})} = S_A + \lambda\psi_A + i\mu\bar{\phi}\chi_A, \\ \bar{S}_{A(\text{new})} = \bar{S}_A + i\left(\frac{7}{4\sqrt{2}} - \mu\right)\bar{\phi}\chi_A + \left(\frac{5}{4\sqrt{6}} - \lambda\right)\bar{\psi}_A, \quad (5.25)$$

where S_A, \bar{S}_A are the orderings used above. Proceeding to solve these constraints, we will find another eight equations. We are only interested in the four equations between B, C and D :

$$(1 + \phi\bar{\phi})\frac{\partial B}{\partial\phi} + \left(\frac{1}{2} - \sqrt{2}\mu\right)\bar{\phi}B + \frac{a}{4\sqrt{3}}\frac{\partial C}{\partial a} - \frac{7}{4\sqrt{3}}C - \frac{\lambda}{\sqrt{2}}C + \frac{\sqrt{3}}{2}\sigma^2 a^2 C = 0, \quad (5.26a)$$

$$\frac{a}{\sqrt{3}}\frac{\partial D}{\partial a} + 2\sqrt{3}\sigma^2 a^2 D - (\sqrt{3} + 2\sqrt{2}\lambda)D - (1 + \phi\bar{\phi})\frac{\partial C}{\partial\phi} - \frac{3}{2}\bar{\phi}C + \sqrt{2}\mu\bar{\phi}C = 0, \quad (5.26b)$$

$$\frac{a}{\sqrt{3}}\frac{\partial B}{\partial a} - 2\sqrt{3}\sigma^2 a^2 B - \sqrt{3}B + \sqrt{2}\left(\frac{5}{2\sqrt{6}} - 2\lambda\right)B$$

$$+(1 + \phi\bar{\phi})\frac{\partial C}{\partial\phi} + \frac{3}{2}\phi C - \sqrt{2}\left(\frac{7}{4\sqrt{2}} - \mu\right)\phi C = 0, \quad (5.26c)$$

$$(1 + \phi\bar{\phi})\frac{\partial D}{\partial\phi} + \frac{1}{2}\phi D - \frac{1}{\sqrt{2}}\left(\frac{7}{2\sqrt{2}} - 2\mu\right)\phi D - \frac{a}{4\sqrt{3}}\frac{ptC}{\partial a} + \frac{7}{4\sqrt{3}}C - \frac{1}{\sqrt{2}}\left(\frac{5}{4\sqrt{6}} - \lambda\right)C + \frac{\sqrt{3}}{2}\sigma^2 a^2 C = 0. \quad (5.26d)$$

We must set $\mu = 7/(8\sqrt{2})$ in order to get rid of $\bar{\phi}C$ in (5.26b) and (5.26c) consistently, and the only freedom left to get consistent equations for \tilde{C} is from λ . By setting $B = \tilde{B}(1 + \phi\bar{\phi})^{\frac{3}{8}}$, $C = \frac{\tilde{C}}{\sqrt{3}}(1 + \phi\bar{\phi})^{-\frac{5}{8}}$, $D = \tilde{D}(1 + \phi\bar{\phi})^{\frac{3}{8}}$, we can again get two decoupled equations for \tilde{C} from the above 4 equations. Again the coefficient of $\sigma^2 a^2 \tilde{C}$ for one equation is $-\frac{7}{4}$ and the coefficient of $\sigma^2 a^2 \tilde{C}$ for the other equation is $\frac{17}{4}$. Hence, we are led again to $\tilde{C} = 0$, showing that the two most interesting orderings give $\tilde{C} = 0$.

VI. Discussion and Conclusion

We would like to compare our results to the ones in Ref. [5]. There, R^2 was used as Kähler manifold and a different result for \tilde{C} was obtained. We would like to investigate here whether we can get a similar result for \tilde{C} in our model in the case of the R^2 Kähler manifold. The Kähler potential would be just $\phi\bar{\phi}$, the Kähler metric is $g_{\phi\bar{\phi}} = 1$ and the Levi-Cita connections are zero. Repeating the steps described in sections II–V, we find out that the structure of the supersymmetry constraints are the same for these two Kähler manifolds. The reason is that the Kähler metric and the connection only enter the Lagrangian through the spin- $\frac{1}{2}$ field χ_A and no other terms. So, there is only a change in the coefficient of $\bar{\phi}\chi_A\chi^B\frac{\partial}{\partial\chi^B}$ in S_A and the corresponding term in \bar{S}_A , the rest being equivalent to put $\phi\bar{\phi} = 0$ in the necessary coefficients. The supersymmetry constraints are then

$$S_A = -2\sqrt{3}i\chi_A\frac{\partial}{\partial\phi} - a\psi_A\frac{\partial}{\partial a} - 6\sigma^2 a^2\psi_A - \frac{i\sqrt{3}}{2}\bar{\phi}\chi_A\chi^B\frac{\partial}{\partial\chi^B} - \frac{1}{4}\psi_B\psi^B\frac{\partial}{\partial\psi^A} - \frac{i\sqrt{3}}{2}\bar{\phi}\chi_A\psi^B\frac{\partial}{\partial\psi^B} - \frac{5}{2}\chi_A\psi^B\frac{\partial}{\partial\chi^B} + \frac{3}{2}\chi^B\psi_B\frac{\partial}{\partial\chi^A} + \psi_A\chi^B\frac{\partial}{\partial\chi^B} \quad (6.1)$$

and

$$\begin{aligned} \bar{S}_A &= 2\sqrt{3}i\frac{\partial}{\partial\chi^A}\frac{\partial}{\partial\phi} + a\frac{\partial}{\partial a}\frac{\partial}{\partial\psi^A} - 6\sigma^2 a^2\frac{\partial}{\partial\psi^A} \\ &+ \frac{i\sqrt{3}}{2}\phi\frac{\partial}{\partial\chi^A}\frac{\partial}{\partial\chi^B}\chi^B - \frac{1}{4}\epsilon^{BC}\frac{\partial}{\partial\psi^B}\frac{\partial}{\partial\chi^C}\psi^A \\ &- \frac{i\sqrt{3}}{2}\phi\frac{\partial}{\partial\psi^B}\frac{\partial}{\partial\chi^A}\psi^B - \frac{5}{2}\frac{\partial}{\partial\psi^B}\frac{\partial}{\partial\chi^A}\chi^B \\ &- \frac{3}{2}\epsilon^{BC}\frac{\partial}{\partial\psi^B}\frac{\partial}{\partial\chi^C}\chi_A - \frac{\partial}{\partial\psi^A}\frac{\partial}{\partial\chi^B}\chi^B. \quad (6.2) \end{aligned}$$

Solving for $S_A\Psi=0$ and $\bar{S}_A\Psi=0$, we obtain eight equations where the four ones between B , C and D are

$$\frac{\partial B}{\partial\phi} + \frac{1}{2}\bar{\phi}B + \frac{a}{4\sqrt{3}}\frac{\partial C}{\partial a} - \frac{7}{4\sqrt{3}}C + \frac{\sqrt{3}}{2}\sigma^2 a^2 C = 0, \quad (6.3a)$$

$$\frac{a}{\sqrt{3}}\frac{\partial D}{\partial a} + 2\sqrt{3}\sigma^2 a^2 D - \sqrt{3}D - \frac{\partial C}{\partial\phi} - \frac{1}{2}\bar{\phi}C = 0, \quad (6.3b)$$

$$\frac{a}{\sqrt{3}}\frac{\partial B}{\partial a} - 2\sqrt{3}\sigma^2 a^2 B - \sqrt{3}B + \frac{\partial C}{\partial\phi} + \frac{1}{2}\phi C = 0, \quad (6.3c)$$

$$\frac{\partial D}{\partial\phi} + \frac{1}{2}\phi D - \frac{a}{4\sqrt{3}}\frac{\partial C}{\partial a} + \frac{7}{4\sqrt{3}}C + \frac{\sqrt{3}}{2}\sigma^2 a^2 C = 0. \quad (6.3d)$$

After the substitution $B = \tilde{B}\exp(-\frac{1}{2}\phi\bar{\phi})$, $C = \frac{\tilde{C}}{\sqrt{3}}\exp(-\frac{1}{2}\phi\bar{\phi})$ and $D = \tilde{D}\exp(-\frac{1}{2}\phi\bar{\phi})$ [5], the above four equations become

$$\frac{\partial\tilde{B}}{\partial\phi} + \frac{a}{12}\frac{\partial\tilde{C}}{\partial a} - \frac{7}{12}\tilde{C} + \frac{1}{2}\sigma^2 a^2 \tilde{C} = 0, \quad (6.4a)$$

$$\frac{\partial\tilde{D}}{\partial\phi} - \frac{a}{12}\frac{\partial\tilde{C}}{\partial a} + \frac{7}{12}\tilde{C} + \frac{1}{2}\sigma^2 a^2 \tilde{C} = 0, \quad (6.4b)$$

$$\frac{\partial\tilde{C}}{\partial\phi} - a\frac{\partial\tilde{D}}{\partial a} - 6\sigma^2 a^2 \tilde{D} + 3\tilde{D} = 0, \quad (6.4c)$$

$$\frac{\partial\tilde{C}}{\partial\phi} + a\frac{\partial\tilde{B}}{\partial a} - 6\sigma^2 a^2 \tilde{B} - 3\tilde{B} = 0. \quad (6.4d)$$

This set of equations is exactly the same as (5.21a–d) if we put $\phi\bar{\phi} = 0$ in there. So, $\tilde{C} = 0$ does not depend on the value of $\phi\bar{\phi}$. We conclude, therefore, that for R^2 as the Kähler manifold, $\tilde{C} = 0$. These results seem to suggest that whatever Kähler manifold one uses, we reach the same conclusion. The reason for the apparent differences with respect to Ref.[5] may lie in the fact that the model used in Ref.[5] was derived from Ref.[28], while ours comes directly from Ref.[27].

Summarizing our work, in section II the more general theory of $N=1$ supergravity with supermatter was described using a canonical approach [13]. The supersymmetry and gauge constraint generators were also presented. In section III we described the Ansatz for the the gravitational field, spin- $\frac{3}{2}$ field and the gauge vector field V_μ^a as well as the scalar fields and the corresponding fermionic partners. In section IV, after a dimensional reduction, we derived the supersymmetric constraints for our one-dimensional model, where one has assumed a FRW closed geometry. In section V, we solved the supersymmetry constraints for the case of a two-dimensional spherically symmetric Kähler manifold, taking the spin-1 field to be zero as well as its fermionic partner. We found that one of the middle states is missing as $C = 0$, and the other middle states have the simple form $B = g(\bar{\phi})(1 + \phi\bar{\phi})^{-\frac{1}{2}}a^3\exp(3\sigma^2 a^2)$ and $D = f(\phi)(1 +$

$\phi\bar{\phi})^{-\frac{1}{2}}a^3 \exp(-3\sigma^2 a^2)$. However, these are not wormhole states. From [6], the wormhole wavefunction has the form prefactor $\times \exp(-(3\sigma^2 a^2 + 3\sigma^2 a^2 \cosh(\rho)))$, where $\phi = \rho e^{i\theta}$, and such behaviour is not provided by B and D . In order to investigate this issue we repeated our study but for a two-dimensional flat Kähler manifold. Once again, we have $C = 0$. Thus, it seems that the results obtained from the framework presented in Ref.[27] are quite general.

The bosonic and filled fermion states have the simple form $A=f(\bar{\phi}) \exp(-3\sigma^2 a^2)$ and $E=g(\phi) \exp(3\sigma^2 a^2)$, respectively. This corresponds to the Hartle-Hawking solution. It is very puzzling that the wormhole state is missing. A similar problem occurred in Ref.[3]. There, the only bosonic physical state is the wormhole solution but the Hartle-Hawking state was missing. However, if we use a different definition of homogeneity [23], we will get the Hartle-Hawking state as the bosonic state but then the wormhole state is missing. We suspect that similar behaviour is occurring here.

In the future we will extend the framework presented in this paper in two directions: the inclusion of all supermatter fields in the process of solving the supersymmetry constraints, and generalizing the work to the case of a Bianchi-IX universe. It will be interesting to see if the same type of results occur there.

Acknowledgements

Part of this work was presented as a Lecture at the International School-Seminar "Multidimensional Gravity and Cosmology", Yaroslavl, Russia, June 20 – 26, 1994. P.R.L.V.M. is very grateful to Prof. V.N. Melnikov and to the Organizing Committee for that opportunity and for all their kind assistance during his visit to Moscow and Yaroslavl. Stimulating conversations with F. Embacher, S.P. Kim, S.W. Kim, M. Rainer, A. Zhuk as well as to K.A. Bronnikov, M. Fil'chenkov, V.D. Ivashchuk, A.A. Kirillov for providing a friendly and delightful atmosphere are also acknowledged. A.D.Y.C. thanks the Croucher Foundation of Hong Kong for financial support. P.R.L.V.M. also gratefully acknowledges the support of a Human Capital and Mobility Fellowship from the European Union (Program ERB4001GT930714).

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