Gravitation & Cosmology, Vol.1 (1995), No.1, pp.22–24

NEUTRINO OSCILLATIONS IN TWISTING MAGNETIC FIELDS

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Received 29 September 1994
Received in final form 25 November 1994

The analysis [1, 2] of neutrino conversion and oscillations between two neutrino species induced by strong magnetic fields is generalized for the particular case of twisting magnetic fields. It is shown that the critical value of the magnetic field strength $\tilde{B}_{\rm cr}$ as a function of the characteristics of neutrinos in vacuum (Δm_ν^2, θ), neutrino magnetic (transition) moment $\tilde{\mu}$ and energy E, the effective particle density of matter $n_{\rm eff}$, and also accounting for the variation of magnetic field along the neutrino trajectory (twisting magnetic field), can be much lower than that for the case of a fixed magnetic field.

In our previous studies [1, 2] we have introduced the critical magnetic field strength $B_{\rm cr}(\Delta m_\nu^2,\ \theta,\ n_{\rm eff},\ E)$ (where $\Delta m_\nu^2 = m_2^2 - m_1^2$, m_1 and m_2 are the two neutrino masses, θ is the vacuum mixing angle, $n_{\rm eff}$ is the effective particle density of matter, $\tilde{\mu}$ is the neutrino magnetic or transition moment, E is the neutrino energy) that determines the range of fields ($B \geq B_{\rm cr}$) for which the magnetic field induced neutrino conversion and oscillations become important. Some applications to neutrinos in magnetic fields of neutron stars, supernovae, the Sun and also neutrinos in interstellar galactic magnetic fields have been discussed. In particular, we have predicted the existance of the "cross-boundary effect" (CBE) that leads to a factor of two decrease in amount of initially emitted left-handed electron neutrinos ν_{e_L} in the bunch as a result of conversion of ν_{e_L} to another neutrino specie (for example, to ν_{e_R}) and subsequent neutrino oscillations induced by strong magnetic field. One of our conclusions that has come from these studies of neutrino conversion and oscillations in different environments is that the influence of magnetic fields becomes important if the field strength is rather high ($B \geq 10^{12} \div 10^{14}\ G$) and/or the magnetic (transition) moment of neutrinos is big ($\tilde{\mu} \sim 10^{-10} \div 10^{-11} \mu_B$). In [1, 2] we have not taken into account the possibility of a change of the transverse magnetic field along the trajectory of neutrinos. However, the twisting magnetic field are of physical interest and can have an important impact, for example, on Solar and supernova neutrinos (see, for example, [3]).

In this paper we generalize the analysis of [1, 2] for the case of twisting magnetic fields and show that the account for the field variation can substantially decrease the critical strength of magnetic fields ($\tilde{B}_{\rm cr}$). From this it follows that the magnetic field induced neutrino conversion and oscillations in the case of twisting field may become important for much smaller values of magnetic field strengths and/or neutrino magnetic (transition) moments than it is in the case of a fixed field. We again restrict our consideration, as in [1, 2], to the case of two neutrino flavours, ν_e and ν_μ. In vacuum the flavour eigenstates ν_e and ν_μ can be expressed in terms of the mass eigenstates ν_1 and ν_2:

$$\begin{aligned} \nu_e &= \ \ \nu_1 \cos\theta + \nu_2 \sin\theta, \\ \nu_\mu &= -\nu_1 \sin\theta + \nu_2 \cos\theta, \end{aligned} \qquad (1)$$

where θ denotes the vacuum mixing angle.

The evolution of neutrinos propagating in matter and transverse twisting magnetic field $\vec{B} = \vec{B}_\perp e^{i\phi(t)}$, (the angle $\phi(t)$ defines the direction of the field in the plane orthogonal to the neutrino momentum) is described by the Schrödinger-type equation

$$i\frac{d}{dt}\nu(t) = H\nu(t), \qquad (2)$$

where the Hamiltonian H can be expressed as a sum of the four terms

$$H = H_V + H_{int} + H_F + H_{\dot{\phi}}. \qquad (3)$$

Here H_V, H_{int} and H_F contain contributions from, respectively, a vacuum mass matrix and neutrino interactions with matter and the magnetic field and the last term $H_{\dot{\phi}}$ accounts for the effect of rotation (twisting) of the magnetic field.

If for the case of Dirac neutrinos one uses the bases in which neutrinos have a definite projection along the direction of propagation

$$\nu = (\nu_{e_L}, \nu_{\mu_L}, \nu_{e_R}, \nu_{\mu_R}),$$

then the Hamiltonian is given by

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$$H^D = \begin{pmatrix} V_- & \Delta m_\nu^2 s/(4E) & \mu_{ee}B & \mu_{e\mu}B \\ \Delta m_\nu^2 s/(4E) & V_+ & \mu_{\mu e}B & \mu_{\mu\mu}B \\ \mu_{ee}B & \mu_{\mu e}B & -\Delta m_\nu^2/(4E) + \dot{\phi}/2 & 0 \\ \mu_{e\mu}B & \mu_{\mu\mu}B & 0 & \Delta m_\nu^2/(4E) + \dot{\phi}/2 \end{pmatrix}, \tag{4}$$

$$V_- = -(\Delta m_\nu^2)/(4E)c + V_{\nu_e} - \dot{\phi}/2, \quad V_+ = (\Delta m_\nu^2)/(4E)c + V_{\nu_\mu} - \dot{\phi}/2,$$

$$V_{\nu_e} = \sqrt{2}G_F\Big(n_e - \frac{1}{2}n_n\Big), \quad V_{\nu_\mu} = -1/\sqrt{2}G_F n_n \tag{5}$$

where n_e and n_n are the electron and neutron number densities. The Hamiltonian (4) corresponds to the case of sterile neutrinos ν_{eR} and $\nu_{\mu R}$.

For the two Majorana neutrinos in the bases written as $\nu = (\nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu)$ in the corresponding Hamiltonian

$$H^M = \begin{pmatrix} V_- & \Delta m_\nu^2 s/(4E) & 0 & \mu B \\ \Delta m_\nu^2 s/(4E) & V_+ & -\mu B & 0 \\ 0 & -\mu B & V_- + \dot{\phi} - 2V_{\nu_e} & \Delta m_\nu^2 s/(4E) \\ \mu B & 0 & \Delta m_\nu^2 s/(4E) & V_+ + \dot{\phi} - 2V_{\nu_\mu} \end{pmatrix}. \tag{6}$$

μ denotes the flavour transition magnetic moment.

Using these Hamiltonians, we can consider different neutrino conversion processes $\nu_i \to \nu_j$ and the corresponding neutrino oscillations $\nu_i \leftrightarrow \nu_j$ induced by the magnetic field such as

$$\nu_{e_L} \to \nu_{e_R},\ \nu_{e_L} \to \nu_{\mu_R},\ \nu_{e_L} \to \bar{\nu}_{\mu_R}. \tag{7}$$

The probability of neutrino conversion from the type j (ν_j) to the type i (ν_i) in passing a distance x in matter and twisting magnetic field is

$$P(\nu_i \to \nu_j) = \sin^2 2\theta_{\text{eff}} \sin^2\Big(\frac{\pi x}{L_{\text{eff}}}\Big), \quad i \neq j, \tag{8}$$

while the survival probability is

$$P(\nu_i \to \nu_i) = 1 - P(\nu_i \to \nu_j), \tag{9}$$

where the effective mixing angle θ_{eff} and the effective oscillation length L_{eff} are given by

$$\tan 2\theta_{\text{eff}} = \frac{2\tilde{\mu}B}{(\Delta m_\nu^2/2E)A - \sqrt{2}G_F n_{\text{eff}} + \dot{\phi}}, \tag{10}$$

$$L_{\text{eff}} = 2\pi\Big[\Big(\frac{\Delta m_\nu^2}{2E}A - \sqrt{2}G_F n_{\text{eff}} + \dot{\phi}\Big)^2 + (2\tilde{\mu}B)^2\Big]^{-\frac{1}{2}}. \tag{11}$$

Note that the effective mixing angle θ_{eff} and the effective oscillation length L_{eff} depend on the magnetic field rotation characteristic $\dot{\phi}$ along the neutrino path (see also [3, 4, 5]).

For different neutrino conversion processes (7) $\tilde{\mu}$, A and n_{eff} are equal to

$$\tilde{\mu} = \begin{cases} \mu_{ee} & \text{for } \nu_{e_L} \to \nu_{e_R} \\ \mu_{e\mu} & \text{for } \nu_{e_L} \to \nu_{\mu_R} \\ \mu & \text{for } \nu_{e_L} \to \bar{\nu}_{\mu_R} \end{cases}, \tag{12}$$

$$A = \begin{cases} \frac{1}{2}(\cos 2\theta - 1) & \text{for } \nu_{e_L} \to \nu_{e_R} \\ \frac{1}{2}(\cos 2\theta + 1) & \text{for } \nu_{e_L} \to \nu_{\mu_R} \\ \cos 2\theta & \text{for } \nu_{e_L} \to \bar{\nu}_{\mu_R} \end{cases}, \tag{13}$$

$$n_{\text{eff}} = \begin{cases} n_e - n_n & \text{for } \nu_{e_L} \to \bar{\nu}_{\mu_R} \\ n_e - \frac{1}{2}n_n & \text{for } \nu_{e_L} \to \nu_{e_R,\mu_R} \end{cases} \tag{14}$$

As it was in the case of a non-twisting magnetic field [1, 2], the probability (10) may have a considerable value (the neutrino conversion processes and oscillations become important) if the following two conditions are valid:

(i) the "oscillation amplitude" $\sin^2 2\theta_{\text{eff}}$ is far from zero (or $\sin^2 2\theta_{\text{eff}} \sim 1$), and

(ii) the length x of the neutrinos path in the medium must be greater than the effective oscillation length L_{eff} ($x \sim$ or $> L_{\text{eff}}/2$).

The condition (i) is realized if $\tan 2\theta_{\text{eff}} \geq 1$, then from (10) it follows that at least one of the following two relations must be satisfied:

$$\text{(a)} \quad (\Delta m_\nu^2/2E)A - \sqrt{2}G_F n_{\text{eff}} + \dot{\phi} = 0 \quad (\tilde{\mu}B \neq 0); \qquad \text{(b)} \quad 2\tilde{\mu}B \geq \Big|(\Delta m_\nu^2/2E)A - \sqrt{2}G_F n_{\text{eff}} + \dot{\phi}\Big|. \tag{15}$$

Let us consider the relation (15b) and assume that the right-hand side is nonzero. By analogy with the case of a non-twisting magnetic field [1, 2], from (15b) we determine the critical magnetic field strength

$$\tilde{B}_{\rm cr} = \left| \frac{1}{2\tilde{\mu}} \Big(\frac{\Delta m_\nu^2 A}{2E} - \sqrt{2} G_F n_{\rm eff} + \dot{\phi} \Big) \right| \quad (16)$$

that constrains the range ($B \geq \tilde{B}_{\rm cr}$) of field strengths for which the value of $\sin^2 2\theta_{\rm eff}$ is not small (i.e., at least is not less than $\frac{1}{2}$) for all possible values of the right-hand side term in (15b).

It is also possible to express $\tilde{B}_{\rm cr}$ in a form more convenient for numerical estimation :

$$\tilde{B}_{\rm cr} \approx 43 \Big(\frac{\mu_B}{\tilde{\mu}} \Big) \Big| - \Big(2.5 \frac{n_{\rm eff}}{10^{31} cm^{-3}} \Big) + A \Big(\frac{\Delta m_\nu^2}{eV^2} \Big) \Big(\frac{MeV}{E_\nu} \Big) + 2.5 \Big(\frac{1m}{L_{\dot{\phi}}} \Big) \Big| \ [\text{Gauss}], \quad (17)$$

where $L_{\dot{\phi}} = 2\pi/\dot{\phi}$.

For the case of strong magnetic fields ($B > \tilde{B}_{\rm cr}$), $\sin^2 2\theta_{\rm eff} \approx 1$, we find that for large enough lengths of a neutrino ν_i path given by $x \approx L_{\rm eff} k/2$, $k = 1, 2, \ldots$ in the magnetized medium characterized by $n_{\rm eff}$ the probability (8) of the conversion process $\nu_i \to \nu_j$ can reach a value of the order of $P(\nu_i \to \nu_j) \sim 1$.

Therefore, an initially emitted, for example, left-handed neutrino can undergo on the path length $x \geq L_{\rm eff}/2$ a conversion to a right-handed (anti)neutrino. These oscillation processes obviously take place only in the presence of strong magnetic fields $B \gg \tilde{B}_{\rm cr}$ and the oscillation length $L_{\rm eff}$, as follows from (11), is $L_{\rm eff} \approx L_F = \pi/\tilde{\mu}B$. For $B \ll \tilde{B}_{\rm cr}$ the magnetic field influence is unimportant and oscillations (if they exist) are completely determined by the vacuum mixing angle and neutrino interaction with matter.

Now let us consider the possibility of twisting magnetic field induced neutrino conversion and oscillations (for definiteness we choose the process $\nu_{e_L} \to \nu_{e_R}$) in the Sun's convective zone. We first estimate the critical field strength using Eqs.(16) or (17) with the following values of neutrino and matter characteristics: $\Delta m_\nu^2 = 10^{-4}\ eV^2$, $\sin 2\theta = 0.1$, $E_\nu = 20$ MeV, $n_{\rm eff} \sim n_e \approx 10^{23}$ cm^{-3}. We assume for the field variation in the convective zone along the neutrino path that $\dot{\phi} > 0$ and use the estimate of Refs.[3,5]: $L_{\dot{\phi}} \sim 0.1 R_\odot \approx 7 \times 10^7$ m, $R_\odot = 7 \times 10^8$ m is the solar radius. Substituting all that to the three terms of (17), we get

$$\tilde{B}_{\rm cr} \approx \Big(\frac{\mu_B}{\tilde{\mu}} \Big) \Big| - 10^{-6} - 5 \times 10^{-7} + 1.43 \times 10^{-6} \Big| \ G = 7 \times 10^{-8} \Big(\frac{\mu_B}{\tilde{\mu}} \Big) \ G. \quad (18)$$

Thus it is obvious that taking into account the magnetic field twisting reduces the critical field value $\tilde{B}_{\rm cr}$ to the order of 7% of the value $B_{\rm cr}$ corresponding to the case of a non-twisting field.

It is supposed that a typical magnetic field value in the convective zone is of the order of $B_{\rm con} \sim 10^5\ G$. It follows from (18) that $B_{\rm con}$ exceeds $\tilde{B}_{\rm cr}$ if the neutrino magnetic (transition) moment exceeds $\tilde{\mu} \geq 10^{-12} \mu_B$.

According to the second condition for the magnetic field induced neutrino conversion and oscillations to become important, the effective oscillation length $L_{\rm eff}$ (Eq.(11)) must be of the order or less than the depth of the convective zone $L_{\rm eff} \leq \frac{1}{2} L_{cz}$. This last condition holds if $\tilde{\mu} \sim 10^{-11} \mu_B$ for the magnetic fields in the convective zone $B \sim 10^5\ G$.

We can conclude that the variation (twisting) of the magnetic field along the neutrino path in the solar convective zone relaxes the critical field strength $\tilde{B}_{\rm cr}$ to the values which can be relevant for the stimulation of a visiable neutrino conversion and oscillations if the neutrino magnetic moment is of the order of $\tilde{\mu} \sim 10^{-11} \mu_B$.

There is also an important increase in the magnetic field induced neutrino conversion and oscillations as compared to what have been discussed in Refs.[1, 2] when one takes into account the possible field twisting in supernovae and neutron stars. The phenomenological implications of this effect for neutrinos in supernovae and neutron stars will be considered elsewhere.

Now, continuing the discussions of Refs [1,2], we should like to mention that the CBE can take place not only at the surface of the neutron star (when neutrinos escape the neutron star matter and start their travel in the empty space where particle number densities $n_e,\ n_n,\ n_p \to 0$). The CBE can effectively appear for the Majorana neutrinos passing through inner layers of the neutron star composed of silicon, oxygen, nitrogen, carbon and helium for these shells $n_{\rm eff} = n_e - n_n \to 0$. Due to the CBE in the inner layers of the neutron star a reasonable amount of active neutrinos can be converted to the sterile (non-interacting with matter) neutrinos able to cause changes in the process of neutron stars cooling.

Acknowledgement

This work was supported in part by the Interregional Centre for Advanced Studies (Moscow, Russia).

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