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CASIMIR-TYPE NULL EXPERIMENT FOR OBTAINING STRONGER RESTRICTIONS ON CONSTANTS OF LONG-RANGE INTERACTIONS

M.Bordag,† V.M.Mostepanenko and I.Yu.Sokolov‡

† *Leipzig University, Institute for Theoretical Physics, Augustusplatz 10, 04109 Leipzig, Germany*[1]

‡ *St.Petersburg State Technological Institute, Dept. of Mathematics Moskovsky pr. 26, 198013 St.Petersburg, Russia*[2]

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A realistic null experiment is suggested in which the Casimir force between a plane plate and a spherical lens is compensated by the force of gravitational attraction. This configuration is shown to be very sensitive to the existence of additional hypothetical forces of Yukawa-type or power laws. From the suggested null experiment the restrictions on the Yukawa constant α can be strengthened by a factor of up to 1000 in a wide range $10^{-8}\mathrm{m} < \lambda < 10^{-4}\mathrm{m}$ and by a factor of 10 for λ from several centimeters to several meters. For power-law interactions the strengthening of restrictions by a factor of 20 is possible for a force decreasing as r^{-5}.

1. Introduction

Recently great interest has been expressed to the search for new long-range interactions and restrictions on their parameters [1-7]. This interest is due to both the possible existence of corrections to the classical gravitational theory at small distances and hypothetical forces between macrobodies resulting from the exchange by light elementary particles between separate atoms. Massless or light elementary particles, such as arion, dilaton, scalar axion, spin-1 antigraviton etc [3,8] are predicted in almost any unified gauge theory, supersymmetry and supergravity. It seems impossible to eliminate such objects from the formalism of modern elementary particle theory. So, a number of attempts were undertaken to observe additional forces in gravitational experiments [2], in elementary particle physics [8] and in astrophysical observations [3].

The strongest restrictions on the constants of the hypothetical forces are obtained in laboratory experiments of Eötvös-type, of Cavendish-type, in measuring Van der Waals and Casimir forces and the transition probabilities in exotic atoms [2,4,5,9,10]. In Ref.[7] the optimal configuration of two test bodies was found, which implied the most tight restrictions on hypothetical forces from force measuring or measuring deviations from the known force law. It was shown that the optimal configuration was very close to the configuration of two plane parallel plates.

[1]e-mail: bordag@qft.physik.uni-leipzig.d400.de
[2]e-mail: mostep@spti.spb.su

One of the most interesting experiments for obtaining the restrictions on hypothetical forces is the experiment on Casimir force measurements. From this experiment the best restrictions on Yukawa-type interactions, described by the potential

$$V(r) = \frac{\alpha}{r} N^2 \exp\left(-\frac{r}{\lambda}\right) , \tag{1}$$

were obtained [10] for the wide range $10^{-8}\mathrm{m} < \lambda < 10^{-4}\mathrm{m}$ (r is the distance between atoms, α is the interaction constant, $\hbar$=c=1, N is the number of nucleons in an atomic nucleus (it is introduced in order to make α independent of the sort of atoms).

Before 1990 the restrictions on the power law interactions, described by the potential

$$V_n(r) = \frac{\lambda_n}{r} N^2 \left(\frac{r_0}{r}\right)^{n-1} , \tag{2}$$

obtained from the Casimir force measurements, also were the best for $n = 2, 3$ [11] (here λ_n are dimensionless interaction constants and $r_0 \equiv 10^{-15}\mathrm{m}$ is introduced for proper dimensionality of V_n). However, in 1990 these restrictions were overcome [12] by the data from a new Cavendish-type experiment [13].

In the present paper a new Casimir-type compensation (null) experiment is suggested for obtaining more severe restrictions on hypothetical interactions. It uses the optimal configurations of Ref.[7] with three, instead of two, test bodies for which the Casimir force is compensated by the gravitational one. This experiment is more sensitive to the presence of additional interactions because it combines the known advantages of compensation experiments with new methods of equilibrium state fixation developed in the recent years in connection with atomic force microscopy. Let us remark that the gravitational compensation scheme is used here in order to give a definite example. Other compensation schemes may eventually play the same role. The suggested experiment allows one to strengthen all known the restrictions on Yukawa type interactions known by now by factors up to 10^3 in the range $10^{-8}\text{m} < \lambda < 10^{-4}\text{m}$ and up to 10 for the λ range from several centimeters to several meters. For power-law interactions the possible strengthening for $n = 4$ is 20 times as compared with the best known results [7]. So the suggested compensation experiment promises improvement of the restrictions for Yukawa-type interactions and for power law interactions of the same order as the optimal experiments with two test bodies of the paper [7]. However, this experiment is much more convenient for experimental technology and deals only with the equipment and configurations used before in the experiments for measuring Van der Waals, Casimir and gravitational forces. The aim of this paper is to attract the attention of experimentalists to this new laboratory experiments on fundamental interactions. The experiments used in Ref.[10] for obtaining restrictions on long range interactions were performed more than 20 years ago. After that a number of new technological possibilities appeared. This justifies the expectations of a significant improvement in the restrictions under consideration.

2. The General Scheme of the Null Experiment

Let us consider a round plate A with area S, thickness H_p and density ρ_3 which is suspended to a point P by a long thread (see Fig.1). A spherical lens made, e.g., of quartz with curvature radius R, height H_l and density ρ_2 is glued to the plate at its center. The lens plays the role of one of the test bodies of the Casimir-type optimal experiment in Ref. [7]. The role of the second test body is played by a large round plate B of thickness D_1 and density ρ_4, which is parallel to A and placed at a distance $0.1\mu\text{m} < d_s < 1.5\mu\text{m}$ from the lens. The configuration of spherical lens over a plane plate was used for the Casimir force measurements in Refs.[14,15]. Due to the large value of R, the Casimir interaction between the lens and the plate B does not differ essentially from the case of two plane parallel plates investigated in [7]. The gravitational interaction between A and B can be neglected.

In order to compensate the attractive Casimir force acting upon A, we use the gravitational interaction between the plate B and the plate C, so that $d_b \gg d_s$, d_b being the distance between A and C (see Fig. 1). The diameter of the plate C is denoted by $2L$, its density by ρ_1, and its thickness by $D >> D_1$. The distance d_b is chosen to be so large that only the gravitational interaction between the plates A and C must be taken into account. The condition $S \ll L^2$ is suggested, so that the plate A is concentrated near the center of C. The value of the gravitational force between the plates A and C can be changed by the variation of D (for example, a special tank with a liquid can be used as C with a mobile back wall for changing the liquid mass and, as a consequence, the gravitational force). For the appropriate range of hypothetical interaction it is more convenient to treat the gravitational force as a probe for hypothetical interactions and the Casimir force as the compensating one.

Figure 1: The general scheme of the null experiment. The plate A of area S and thickness H_p has the density ρ_3 and the density of nucleons n_3. The spherical lens of height H_l and curvature radius R has the density ρ_2 and the nucleon density n_2. Its distance from the plate B is d_s. The thickness of B is D_1 and the densities are ρ_4 and n_4. The plate C has the diameter $2L$, other parameters D, ρ_1, n_1 and its distance from A is d_b.

Let us now find the equilibrium conditions between the Casimir and gravitational forces acting on the plate A with the lens. The Casimir force between the plane plate B and the spherical lens is given by the formula [14]

$$F_{\text{Cas}} = \frac{2\pi R}{3}\frac{B_c}{d_s^3}\,, \qquad (3)$$

where $B_c \approx 3 \times 10^{-28}\text{Jm}$ for the metallic plate and quartz lens, B_c is calculated with the relative error of 10-20% (we can neglect the Casimir interaction between B and the underlying plate A due to the rapid decrease of the Casimir force with distance).

To find the gravitational force between the plate C and the plate A with the lens let us calculate firstly the force between C and an individual atom of mass

m_a at a distance h from the center of C. It is

$$F_{g,a} = 2\pi\rho_1 G \int_0^L x\mathrm{d}x \int_{-D}^0 \mathrm{d}z \frac{m_a}{\sqrt{x^2+(z-h)^2}} = 2\pi\rho_1 Gm\Big[\frac{D}{L} + \sqrt{1+(h/L)^2} - \sqrt{1+((D+h)/L)^2}\Big], \quad (4)$$

where G is the Newtonian gravitational constant.

Integrating this equation over the volumes of the plate A and the spherical lens, we obtain for the sought force in the zero order in the small parameters d_b/L, D/L, H_p/L, $\sqrt{S}/L$:

$$F_g = 2\pi\rho_1 G\big[SDH_p\rho_3 + \frac{\pi\rho_2}{3}\left(R - \frac{H_l}{3}\right)DH_l^2\big]. \quad (5)$$

For numerical estimations it is convenient to rewrite the results (3), (5) in terms of the characteristic values of all these parameters:

$$F_{\mathrm{Cas}} = 4\times 10^{-12}N \times\Big(\frac{R}{0.7\mathrm{cm}}\Big)\Big(\frac{1\mu\mathrm{m}}{d_s}\Big)^3\Big(\frac{B_c}{3\times 10^{-28}\mathrm{Jm}}\Big), \quad (6)$$

$$F_g = 4\times 10^{-12}N\Big(\frac{\rho_1}{10^3\mathrm{kg/m}^3}\Big) \times\Big(\frac{\rho_3}{2\times 10^3\mathrm{kg/m}^3}\Big)\Big(\frac{D}{4\mathrm{cm}}\Big)\Big(\frac{S}{1\mathrm{cm}^2}\Big)\Big(\frac{H_p}{1\mathrm{mm}}\Big) \quad (7)$$

where it is also taken into account that the second term in (5), which describes the contribution of the lens, is small compared with the first one.

Now it is necessary to discuss the accuracy with which the equality of F_{Cas} and F_g can be obtained. The value of the gravitational constant G is known up to 1% [16]. So it is desirable to know the value of B_c with the same accuracy. However, the theoretical value of B_c presented in the beginning of this section is not so accurate. For this purpose it is necessary to state experimentally the equilibrium of the pendulum system under consideration with certain values of D and d_s (the gravitational forces (5) do not depend on d_b in our approximation) and then to calculate the value of B_c. After that one can use the obtained value of B_c with a relative error of 1% for the calculations of restrictions on hypothetical interactions for the other values of D and d_s.

The suggested scheme of a null experiment may be said to be a composition of experimental devices previously used in the measurements of Casimir and gravitational forces. For example, all the experimental difficulties in measuring the Casimir forces (nonperfectness of test body surfaces, measuring of small distances, different types of noises connected with measurement of small forces, etc.) can be overcome like that was done in the papers [14,15,17]. Then, extremely small gravitational forces (up to 2×10^{-15}N, that is, by 3 orders less than in [7]) were detected successfully in Ref.[18]. In that paper a torsion balance was used to achieve such a high sensitivity. However, as we shall see below, the force sensitivity of the order of 10^{-13}N can be achieved with the help of the pendulum scheme under consideration in this paper. Lastly, the compensation scheme of the experiment is generally used for obtaining the sensitivity limits of different physical quantities (see, e.g., [19]).

So the Casimir force F_{Cas} can be compensated by the gravitational force F_g with a relative error of 1%. If the additional hypothetical interaction exists between the bodies A, B and A, C, then the equilibrium state may be broken due to the difference of additional interactions between A, B and A, C. Let us discuss separately the cases of Yukawa and power law hypothetical interactions and analyze, for which parameters values the strongest restrictions on hypothetical interaction constants could be obtained if no equilibrium break were observed.

3. Restrictions on Yukawa-type Hypothetical Interactions

As the hypothetical interaction is very small, it can be obtained by additive integration of the potential (1) over the volumes of the test bodies. Let us calculate firstly the force between the plate C and an atom at a distance h from the center of C due to the potential (1). By simple integration we obtain:

$$F_{H,a}(h) = 2\pi\alpha\lambda n_1 N^2\Big[\mathrm{e}^{-h/\lambda} - \mathrm{e}^{-(D+h)/\lambda} + \mathrm{e}^{-\sqrt{L^2+(D+h)^2}/\lambda} - \mathrm{e}^{-\sqrt{L^2+h^2}/\lambda}\Big] \quad (8)$$

where n_1 is the number of atoms per unit volume of the material of the plate C.

From Casimir force measurements the best restrictions on α follow for the range $10^{-8}\mathrm{m} < \lambda < 10^{-4}\mathrm{m}$, hence we can neglect the last two terms in (8). Integrating then the result (8) over the volumes of the plane plate A and the spherical lens, we obtain an expression for the hypothetical force between them and the plate C:

$$F_{H,1} = Sn_3\int_{d_b}^{d_b+H_p} F_{H,a}(h)\,\mathrm{d}h + \pi n_2\int_{d_b+H_p}^{d_b+H_p+H_l} F_{H,a}(h) \big[R^2 - (R-d_b-H_p-H_l+h)^2\big]\,\mathrm{d}h$$
$$= 2\pi\alpha n_1\lambda^2 N^2 \Big\{\mathrm{e}^{-d_b/\lambda}\Big[1-\mathrm{e}^{-D/\lambda} - \mathrm{e}^{-H_p/\lambda} + \mathrm{e}^{-(D+H_p)/\lambda}\Big] n_3 S + \pi n_2 \mathrm{e}^{-(D+H_l+H_p+d_b)/\lambda}\Big(\mathrm{e}^{D/\lambda}-1\Big) \times\Big[\mathrm{e}^{H_l/\lambda}\left(H_l^2 - 2H_lR - 2H_l\lambda + 2R\lambda + 2\lambda^2\right)$$

$$-2R\lambda - 2\lambda^2\Big]\Big\} \qquad (9)$$

where n_3 and n_2 are the numbers of atoms per unit volume of the plate A and the lens materials.

In a quite similar way the hypothetical force between the plate A with the spherical lens and the plate B may be found (taking into account that the characteristic size of the plate B is also of the order of L):

$$\begin{aligned} F_{H,2} &= Sn_3 \int_{d_s+H_l}^{d_s+H_l+H_p} F_{H,a}(h)\Big|_{D=D_1,\ n=n_4} \mathrm{d}h \\ &+ \pi n_2 \int_{d_s}^{d_s+H_l} F_{H,a}(h)\Big|_{D=D_1,\ n=n_4} \mathrm{d}h \\ &\quad \times \left[2R(h-d_s) - (h-d_s)^2\right] \mathrm{d}h \\ &= 2\pi\alpha n_4 \lambda^2 N^2 \Big\{ \mathrm{e}^{-(H_l+d_s)/\lambda} \\ &\quad \times \Big[1 - \mathrm{e}^{-D_1/\lambda} - \mathrm{e}^{-H_p/\lambda} + \mathrm{e}^{-(D_1+H_p)/\lambda}\Big] n_S \\ &+ \pi n_2 \mathrm{e}^{-(D_1+H_l+d_s)/\lambda} \left(\mathrm{e}^{D_1/\lambda} - 1\right) \Big[2H_l R - H_l^2 \\ &- 2H_l\lambda - 2\lambda^2 + 2R\lambda + 2\mathrm{e}^{H_l/\lambda}\lambda(\lambda - R)\Big]\Big\} \qquad (10) \end{aligned}$$

where n_4 is the number of atoms per unit of volume of the plate B.

Now it is possible to obtain restrictions on α depending on λ from the evident inequality

$$|F_{H,1} - F_{H,2}| < F_g/100\,, \qquad (11)$$

which means that no deviations from equilibrium state are registered, possibly caused by the hypothetical force with the value of $\geq 0.01F_g$. As it is seen from (7), the right-hand side of (11) is of the order of $> (10^{-13}...10^{-12})$N. Here we must emphasize that it is unnecessary to measure such forces and only the sensitivity of the used setup to forces shift from zero is needed. Recall that the to-date experimental limit on such a quantity is 2×10^{-15}N [18] and it was achieved by the torsion balances. In Ref.[17] the force sensitivity on the level of 10^{-12}N was achieved by the pendulum scheme. For this purpose the small oscillations (of several angstroms) near the state of equilibrium were used. It is now possible to detect the deviation of a body up to 0.1 from its state of equilibrium by an atomic force microscope tip [20, 21]. Then the quantity of the order of 10^{-13}N may be detected by a pendulum scheme using an autotracking system with a back reaction, which has been successfully applied in atomic force microscopy [20, 21].

The results of numerical calculations based on Eqs.(9)-(11) are presented in Fig.2 by the curve 1. The permitted range of (α,λ) lies below the curve 1. For each λ an optimization was made with respect to the parameters D, S, H_p, H_l, R, d_b to get the strongest restriction on α. For the same purpose the following values for the densities were chosen: $\rho_1 = 10^3\mathrm{kg/m}^3$, $\rho_2 = \rho_3 = 2\times10^3\mathrm{kg/m}^3$, $\rho_4 = 2\times10^4\mathrm{kg/m}^3$. For example, for $\lambda = 10^{-5}$m the optimal parameters values are: $D = 0.11$m, $S = 10^{-3}\mathrm{m}^2$, $H_p = 6.3\times10^{-4}$m, $H_l = 10^{-4}$m, $R = 6.6\times10^{-3}$m, the value of d_b practically does not affect the result in a wide range from 1 cm and more. In the range of $\lambda > 10^{-6}$m the optimal distance d_s for obtaining the strongest restrictions was found to be equal to (1-1.5)μm. Beginning with $\lambda \approx 10^{-6}$m and less, the optimal distance d_s decreases from 1μm to 0.1μm. It should be noted that in the range $10^{-8}\mathrm{m} < \lambda < 0.1$m the role of a probe force to hypothetical interactions is played by the Casimir force and the gravitational force is only a compensation. For the range $\lambda > 0.1$m the situation changes *vice versa*: the gravitational force is a probe and the Casimir force compensates it. In the same figure the curve 2 shows the best restrictions on hypothetical interactions obtained to-date from different laboratory experiments [7].

Figure 2: The restrictions on the Yukawa interaction parameters (α,λ) from: 1 - the suggested Casimir-type null experiment; 2 - all experiments made up to-date.

It is seen that in the wide range $10^{-8}\mathrm{m} < \lambda < 10^{-4}$m (for which the known results obtained from the Casimir experiments of [15,17] are shown by the curve 2) the best restrictions on α can be strengthened in the suggested experiment up to a factor of 1000. In the range between several centimeters and several meters the restrictions on Yukawa-type interactions known to-date may be strengthened up to a factor of 10 (the known results, presented here by the curve 2, follow from Eötvös-type experiments of Ref.[5]).

Table 1: The limits on the constants of power law long-range interactions from the suggested experiment and the values of experimental device parameters

n	$\lambda_{n\ max}$	D(cm)	S(m^2)	H$_p$(m)	H$_l$(m)	R(m)	d$_s$(μm)	$\delta\Delta F = \frac{F_g}{100}$(N)
2	1.6×10^{-29}	12	10^{-4}	10^{-4}	10^{-2}	10^{-2}	1.5	2×10^{-13}
3	1.4×10^{-16}	3.6	10^{-2}	2×10^{-3}	10^{-4}	3.2	1.5	6×10^{-12}
3	1.4×10^{-16}	3.6	10^{-4}	2×10^{-3}	10^{-4}	3×10^{-2}	1.5	6×10^{-14}
4	5 ×10^{-5}	10	10^{-4}	10^{-4}	10^{-4}	9×10^{-2}	1.45	2×10^{-13}

4. Restrictions on Power-Law Hypothetical Interactions

Now let the hypothetical force between separate atoms of macrobodies be due to the potential (2). In a manner similar to Sec.3, we can calculate the additional forces acting between the test bodies A, C and A, B. The hypothetical force acting between the plate C and an atom at a distance h from the center of C is

$$F^n_{H,a} = 2\pi\lambda_n N^2 r_0^{n-1} n_1 \int_0^L x\mathrm{d}x \left\{ \frac{1}{(x^2+h^2)^{n/2}} - \frac{1}{(x^2+(h+D)^2)^{n/2}} \right\} . \tag{12}$$

In the cases n =2,3,4 – for which an improvement of modern restrictions will be possible in the experiment being suggested – the results of integration (12) read:

$$F^{(2)}_{H,a} = 2\pi\lambda_2 N^2 r_o n_1 \left[\ln\frac{D+h}{h} - \frac{1}{2}\ln\frac{(D+h)^2+L^2}{h^2+L^2} \right], \tag{13}$$

$$F^{(3)}_{H,a} = 2\pi\lambda_3 N^2 r_o^2 n_1 \left[\frac{D}{h(D+h)} - \frac{1}{\sqrt{h^2+L^2}} + \frac{1}{\sqrt{(D+h)^2+L^2}} \right], \tag{14}$$

$$F^{(4)}_{H,a} = \pi\lambda_4 N^2 r_o^3 n_1 L^2 \left[\frac{1}{h^2(L^2+h^2)} - \frac{1}{(D+h)^2[(D+h)^2+L^2]} \right]. \tag{15}$$

Now the additional hypothetical forces $F^{(n)}_{H,1}, F^{(n)}_{H,2}$ acting due to the potential (2) between the plate A with the lens and the plates C, B are correspondingly expressed by the first lines of Eqs.(9), (10) in which the function $F_{H,a}(h)$ must be replaced by $F^{(n)}_{H,a}(h)$ (Eqs.(13)-(15)). All these integrals can be calculated in an analytic form in the zero order with respect to the small parameters $d_b/L,\ D/L,\ H_l/L,\ \sqrt{S}/L,\ d_s/L,\ D_1/L$. For the sake of brevity we will write out here only the results for $n = 4$. As it will be seen below, in this case the suggested experiment leads to the most fruitful results for the power law interactions. So the expression for $F^{(4)}_{H,1}$ reads :

$$\begin{aligned} F^{(4)}_{H,1} &= \pi\lambda_4 N^2 r_o^3 n_1 \Bigg\{ S n_3 H_p \left[\frac{1}{d_b(H_p+d_b)} - \frac{1}{(d_b+D)(H_p+d_b+D)} \right] \\ &+ \pi n_2 \Bigg[H_l \left(\frac{D+d_b+H_l+H_p-2R}{D+d_b+H_p} - \frac{d_b+H_l+H_p-2R}{d_b+H_p} \right) \\ &+ 2(d_b+H_l+H_p-R)\ln\frac{d_b+H_l+H_p}{d_b+H_p} 2(D+d_b+H_l+H_p-R)\ln\frac{D+d_b+H_l+H_p}{D+d_b+H_p} \Bigg] \Bigg\}. \end{aligned} \tag{16}$$

The corresponding expression for $F^{(4)}_{H,2}$ is

$$\begin{aligned} F^{(4)}_{H,2} &= \pi\lambda_4 N^2 r_o^3 n_1 \Bigg\{ S n_3 D \left[\frac{1}{(d_s+H_l)(H_l+d_s+D)} - \frac{1}{(d_s+H_l+H_p)(H_p+H_l+d_s+D)} \right] \\ &+ \pi n_2 \Bigg[D_1 + \frac{(d_s+2R)d_s}{d_s+H_l} - \frac{(D_1+d_s)(D_1+d_s+2R)}{D+d_s+H_l} \\ &+ 2(d_s+R)\ln\frac{d_s+H_l}{d_s} - 2(D+d_s+R)\ln\frac{D+d_s+H_l}{D+d_s} \Bigg] \Bigg\} . \end{aligned} \tag{17}$$

Now the restrictions on λ_n may be obtained from the inequality (11) using the expressions (13)-(17) for the hypothetical forces. In all the cases the values of the plates' geometric parameters were chosen to obtain the strongest restrictions. All these results, along with the relevant laboratory setup sensitivity to the force value displacement are collected in Table 1. The optimal densities are the same as in Sec.3. As seen from the table, the greatest improvement of the restrictions on λ_n from the suggested experiment (by a factor of 20) takes place for $n = 4$ (now it is known from Cavendish-type experiments that[12] $\lambda_4 \leq 1 \times 10^{-3}, \lambda_3 \leq 0.7 \times 10^{-16}, \lambda_2 \leq 0.7 \times 10^{-29}$). So the suggested experiment promises a large progress in obtaining the most tight bounds for Yukawa-type hypothetical interactions and power-law ones for $n = 4$. For other power - law interactions it will lead to results comparable with those obtained in the Cavendish-type experiments.

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