

# COMMENTS ON SIGNATURE CHANGE AND THE MULTIDIMENSIONAL WHEELER-DEWITT EQUATION<sup>1</sup>

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It is argued heuristically – using an  $\mathbf{S}^3 \times \mathbf{S}^6$  minisuperspace model – that there might be a fundamental mechanism (based on either quantum gravity or classical signature change) that stabilizes internal spaces with non-vanishing Ricci curvature.

## 1. Introduction

In this contribution, I would like to present a particular speculation in a quite heuristic manner. It concerns the stabilization of scale factors belonging to internal spaces. The underlying physical problem is provided by the typical classical behaviour of scale factors in the very simplest higher-dimensional cosmological models (see e.g. Refs. [1–3]): some blow up, while the others usually collapse, either approaching zero asymptotically or running into a singularity after a finite amount of proper time (“crack of doom”). The papers [1] and [2], as well as the references contained therein provide examples for these two types. Even those rare classical solutions which evolve towards finite values of the internal scale factors are unstable against small perturbations – in contrast to what we observe (clap your hands, and you will certainly not cause an internal space to collapse).

Stabilization of an internal space can be achieved in more sophisticated models in which a particular interaction between gravity and matter may capture some of the scale factors near a (possibly local) minimum of an effective potential. (Maybe the most popular approaches are those inspired by higher-dimensional supergravity theories). For a selection of some papers on this issue, see Refs. [4–9]. Some authors considered an effective  $R^2$ -action for gravity [10], the use of finite temperature quantum field theoretic methods (see e.g. Ref.[11]), or cosmological models based on the theory of superstrings [12]. It is worth, however, thinking about the possible existence of pure gravitational mechanisms causing a comparable effect in a logically and technically simpler way (by “pure” I mean that matter enters a model in a very simple or

even a more or less symbolic form, e.g. as a massive scalar field or a cosmological constant). One such possibility might be provided by the gravitational Casimir effect [13], but this is not very much clear by now.

Another area in which a “pure gravity” solution to the stabilization problem may be looked for is marked by the minisuperspace approaches to quantum cosmology [14–17], based on the (possibly path integral motivated) Wheeler-DeWitt equation and boundary conditions for the wave function of the universe. Concerning these attempts, one must say that only few solutions to few models are known. There is especially one model – maybe the most natural to consider in this context – that is remarkably resistant against analytical methods: a ten-dimensional space-time with  $\mathbf{S}^3 \times \mathbf{S}^6$  as spatial sections, and a positive cosmological constant  $\Lambda$ . (I will definitely refuse to assume the existence of a supergravity-inspired rank six antisymmetric tensor field which would generate stability of the  $\mathbf{S}^6$  by virtue of a Freund-Rubin mechanism [18]; see also Ref. [7]). This model I will talk about, sometimes using  $\mathbf{S}^3 \times \mathbf{T}^6$  for comparison.

By the shorthand notations used above I mean the class of metrics

$$ds^2 = -\mathcal{N}(t)^2 dt^2 + a_1(t)^2 d\sigma_3^2 + a_2(t)^2 d\sigma_6^2 \quad (1)$$

where  $d\sigma_3^2$  is the metric on the unit three-sphere, and  $d\sigma_6^2$  is the metric on the unit six-sphere (or on the flat unit six-torus, in which case the internal dimensions do not contribute a curvature term in the action). This model (and, more general,  $\mathbf{S}^m \times \mathbf{S}^n$  with  $m, n \geq 2$ ) showed up in some of the lists given by other contributors at this conference (e.g. A. Zhuk), and it was implicitly declared “non-fully-integrable” in the sense of not being reducible to an (integrable) Toda system. We shall see that this characterization is possibly connected with physically desirable proper-

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ties. The mathematical problem is that one has to treat the two scale factors  $a_1$  and  $a_2$  on essentially the same footing – there seem to be very few possibilities to get rid of one degree of freedom (e.g. by perturbative approximations). This is in contrast, for example, to the Friedmann-Robertson-Walker (FRW) models with a single scalar field  $\phi$  [15], where, in a first approximation,  $\phi$  appears only through the effective cosmological constant  $V(\phi)$ , and the two dimensions of minisuperspace are reduced to one. As a consequence, the application to (1) of methods developed for simpler models [6, 14, 15, 16, 17] is likely to be problematic, and we are far from knowing the set of all physically reasonable quantum states. Some material on this model may be found in Refs. [19, 8, 9, 7].

In order to develop my speculation, let me outline briefly what one usually does in minisuperspace quantum cosmologies, using (1) as a starting point. (For an excellent introduction to this subject see Ref.[17].) The Einstein-Hilbert action for ten-dimensional gravity, including a (positive) gravitational constant and the usual boundary term that subtracts a divergence, is given by

$$S = -C \int_{\mathcal{M}_{10}} d^{10}x \sqrt{-g} ({}^{10}R + 2\Lambda) - 2C \int_{\partial\mathcal{M}_{10}} d^9x \sqrt{h} K, \quad (2)$$

where

$$C = \frac{m_P^2}{16\pi} (\text{volume of internal space today})^{-1}, \quad (3)$$

$h_{ij}$  is the metric induced by  $g_{\mu\nu}$  on the boundary  $\partial\mathcal{M}_{10}$  and  $K$  is the trace of its extrinsic curvature. The above choice of  $C$  ensures the correct gravitational constant today. Inserting (1), one obtains an action  $S = \int dt \mathcal{L}(a_1, a_2, \dot{a}_1, \dot{a}_2, \mathcal{N})$  that we will not display (see e.g. Ref.[11]), but just mention that the total volume of the  $t = \text{const}$  space sections is given by

$$V = \int_{\mathcal{M}_9} d^9x \sqrt{h} = w a_1^3 a_2^6, \quad (4)$$

where  $w = 32\pi^5/15$  in the  $\mathbf{S}^6$  case (then we define  $k = 1$ ), and  $w = 2^7\pi^8$  in the  $\mathbf{T}^6$  case ( $k = 0$ ).

Next we change variables according to

$$\begin{aligned} N &= \frac{\Lambda}{3\sqrt{10}} a_2 \mathcal{N}, \\ u &= \frac{\Lambda^2}{180} a_1^2 a_2^2, \\ v &= \frac{\Lambda^3}{5400\sqrt{5}} a_1 a_2^5 \end{aligned} \quad (5)$$

and make the definition (using  $\Lambda \equiv \ell_\Lambda^{-2}$  and  $m_P \equiv$

$\ell_P^{-1}$ )

$$\begin{aligned} \Lambda_{\text{eff}} &= \frac{375^{1/4} \Lambda}{2^{1/8} 6 (wC)^{1/4}} \\ &\equiv \left( \frac{1}{\pi} \left( \frac{5}{3\sqrt{2}} \right)^3 \left( \frac{\ell_P}{\ell_\Lambda} \right)^2 \left( \frac{a_2(\text{today})}{\ell_\Lambda} \right)^6 \right)^{1/4}, \end{aligned} \quad (6)$$

the last identity being valid only in the  $k = 1$  case when  $a_2$  compactifies. (Let us remark here that one could consider any  $\mathbf{S}^n$  ( $n \geq 2$ ) instead of  $\mathbf{S}^6$ , but then the prefactors and exponents in these definitions would be even more messy). In these new variables, the action (2) becomes

$$S = \frac{1}{\Lambda_{\text{eff}}^4} \int dt \left( -\frac{\dot{u}\dot{v}}{N} - NW(u, v) \right), \quad (7)$$

where the potential is given by

$$W(u, v) = -v - k u^{3/2} + 2u^{5/4} v^{1/2}. \quad (8)$$

The corresponding Euclidean action (obtained by sending  $t \rightarrow it$ , thus changing (1) into the Riemannian metric  $ds_E^2 = \mathcal{N}(t)^2 dt^2 + a_1(t)^2 d\sigma_3^2 + a_2(t)^2 d\sigma_6^2$ ) reads

$$I = \frac{1}{\Lambda_{\text{eff}}^4} \int dt \left( -\frac{\dot{u}\dot{v}}{N} + NW(u, v) \right). \quad (9)$$

The Wheeler-DeWitt metric reduces to  $ds_{WDW}^2 = -dudv$ , which shows that  $u$  and  $v$  are "lightlike" coordinates in minisuperspace (both ranging from 0 to  $\infty$ ). We define the "timelike" direction in minisuperspace by  $dudv > 0$ . The Hamiltonian corresponding to (7) is the well-known energy constraint

$$\mathcal{H} = N (-p_u p_v + \Lambda_{\text{eff}}^{-8} W(u, v)) \quad (10)$$

that has to vanish (on the constrained phase space or on physical quantum states, respectively). The classical (Lorentzian) solutions of this system must obey (in the gauge  $N = 1$ )

$$\dot{u}\dot{v} = \pm W, \quad \ddot{u} = \pm \partial_v W, \quad \ddot{v} = \pm \partial_u W, \quad (11)$$

with the + signs (the first of these just meaning  $\mathcal{H} = 0$ ); the corresponding Euclidean solutions – following from variation of  $I$  – obey (11) with the "–" signs. Lorentzian and Euclidean solutions (trajectories in minisuperspace) describe ten-dimensional geometries when re-transformed and inserted into (1) and its Euclidean counterpart, respectively. The variables chosen are such that the form of the classical equations of motion is formally independent of  $\Lambda$  and  $C$ .

A transition to quantum mechanics is achieved by replacing  $p_u \rightarrow -i\partial_u$ ,  $p_v \rightarrow -i\partial_v$  in  $\mathcal{H}$ , thus leading to the Wheeler-DeWitt equation

$$\partial_{uv} \psi(u, v) = -\Lambda_{\text{eff}}^{-8} W(u, v) \psi(u, v). \quad (12)$$

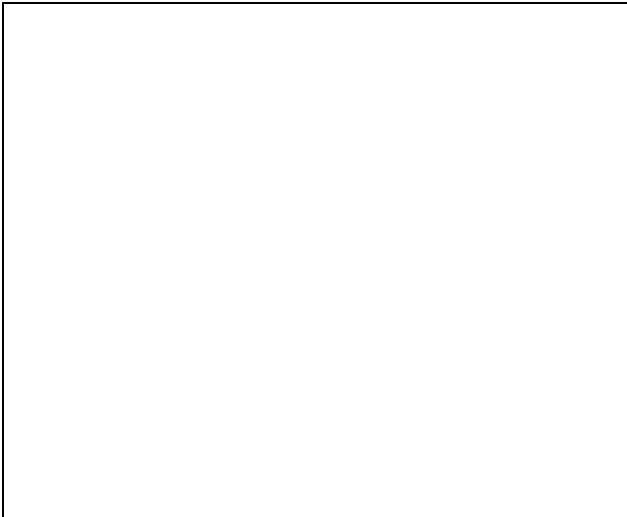


Figure 1: This plot displays, using a  $(u, v)$ -coordinate system of minisuperspace, the zero-potential curve (dashed), with its two asymptotic branches that are called "upper" and "lower" in the text, as well as some classical Euclidean trajectories. The instantons are represented by those two curves which appear to end at  $W = 0$  (they actually have turning points there). The arrows indicate the direction off the regular zero-geometry. Recall that the "timelike" direction in minisuperspace is given by  $dudv > 0$ . This may be considered defining a "light-cone" that is rotated by  $45^\circ$  against the usual one in the special relativistic  $(t, x)$ -diagrams.

The operator ordering has been chosen in the only way that is consistent with general covariance under transformations of all the three variables, including the lapse function [17]. (Let us note in brackets that by choosing other variables  $\bar{u} = \bar{u}(u)$ ,  $\bar{v} = \bar{v}(v)$ , one can achieve various forms of the potential  $\bar{W} = W(du/d\bar{u})(d\bar{v}/dv)$  by transforming the lapse as  $\bar{N} = N(d\bar{u}/du)(d\bar{v}/dv)$ . However, the choice (8) seems to me to be the best one.

Having set up convenient variables, let us now look at some basic features of the model. (Clearly, in view of (11, 12), all such features are contained in the potential  $W$ , because the rest is fairly trivial). First we note that there is a curve  $W = 0$  (which may be found by solving (8) with respect to  $v$ ), lying entirely in the interior of the minisuperspace (see Fig.1).

In the region  $W < 0$  (which extends to the axes  $u = 0$  and  $v = 0$ ), the Euclidean solutions are necessarily timelike curves (with respect to the Wheeler-DeWitt metric), the Lorentzian ones are spacelike curves (cf. the first equation of (11)). The opposite is of course true if  $W > 0$ . An escape of both scale factors  $(a_1, a_2)$  to large values is only possible inside the  $W > 0$  region which one is tempted to call the "classical" one. The region  $W < 0$  could then be called the "quantum" or "Euclidean" regime. Note that the two asymptotic branches of the zero-potential curve

( $v \rightsquigarrow 4u^{5/2}$  for the "upper" and  $v \rightsquigarrow \frac{1}{4}u^{1/2}$  for the "lower" branch in Fig.1) approach constant  $a_1 \rightarrow \ell_\Lambda \sqrt{3}$  and  $a_2 \rightarrow \ell_\Lambda \sqrt{15}$ , respectively (which is, if  $\ell_\Lambda \approx \ell_P$ , what one expects from a classical/quantum transition regime).

Moreover, the curve  $W = 0$  provides all points in which a classical Euclidean solution may be matched smoothly to a Lorentzian one. It is not quite clear which role Euclidean trajectories should play in a sensible quantum cosmology, but in some of the most prominent approaches they provide a key for finding the state of the universe. Euclidean trajectories describing regular ten-geometries emerge from the origin ( $u(0) = v(0) = 0$ ) and behave there like  $v \sim c_1 u^{5/2}$ , then  $a_1(0) = \text{finite}$  or  $v \sim c_2 u^{1/2}$ , then  $a_2(0) = \text{finite}$ .

One usually singles out two particular Euclidean solutions as "instantons": one being along the curve  $v = (9/16)u^{5/2}$  (in  $W < 0$ ) and the other along  $v = (4/9)u^{1/2}$  (in  $W < 0$ , too). They give rise to the Riemannian ten-geometries  $\mathbf{S}^3 \times \mathbf{S}^7$  and  $\mathbf{S}^4 \times \mathbf{S}^6$ , respectively, and are the only Euclidean solutions starting at the origin and having a turning point ( $\dot{u} = \dot{v} = 0$ , i.e. zero extrinsic curvature) at  $W = 0$ . They are usually considered as the preferred candidates describing the coming-into-(classical)existence of the universe by quantum tunneling [7, 8, 9, 19]. In Fig.1, these instantons, as well as several other typical Euclidean trajectories, are displayed.

When expressed in terms of their proper time ( $\mathcal{N} = 1$ ), the Lorentzian solutions either expand in both scale factors  $a_1$  and  $a_2$  exponentially (thus describing inflation without compactification), or expand in one scale factor and contract in the other, thereby entering the "quantum" region  $W < 0$  and finally collapsing towards one of the axes in a Kasner-type singularity ( $a_1 \rightarrow \infty$  while  $a_2 \rightarrow 0$ , or vice versa). The only solutions that compactify are the Lorentzian analogues of the two instantons (lying on the same curves as these, but now in the region  $W > 0$ ), and even these are unstable against small perturbations. Each of these two very special solutions are characterized by one scale factor being actually constant ( $a_1 = \ell_\Lambda \sqrt{8}$  and  $a_2 = \ell_\Lambda \sqrt{20}$ , respectively). In the approaches that connect the instantons to the most probable classical evolution of the universe [7, 8, 9, 19], this last number is interpreted as the actual Kaluza-Klein scale factor. In Fig.2, some Lorentzian trajectories are shown.

Comparing this structure with the  $k = 0$  case, we find that there the zero-potential curve  $W = 0$  is just given by  $v = 4u^{5/2}$ , and the lower asymptotic branch has disappeared.

Can a transition from the classical trajectories satisfying (11) to the Wheeler-DeWitt equation (12) improve likelihood and stability of compactification? Let me adopt as a starting point the prescription of Hartle and Hawking [14] for finding the "no-boundary"-

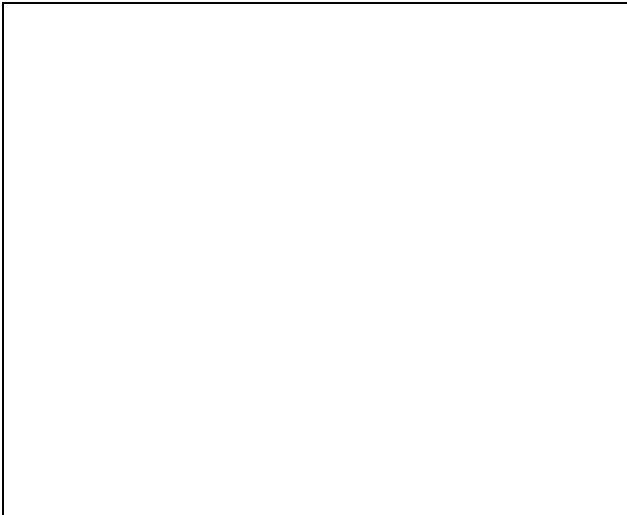


Figure 2: Some Lorentzian (“physical”) trajectories are drawn. The only solutions describing compactification (i.e. the counterparts of the two instantons) are those curves which end at their turning points on the (dashed) zero-potential line. All other trajectories displayed were (without loss of much generality) chosen without turning point and are extended maximally in both directions of their time parameter. Concerning the fact that some of them hit the axes, recall from (5) that, for example,  $v \rightarrow 0$  at finite  $u$  means  $a_1 \rightarrow \infty$  while  $a_2 \rightarrow 0$ . This behaviour is referred to as “Kasner-type” singularity in the text. A nice example is provided by the trajectory starting from  $(u \approx 6.79, v = 0)$ , evolving into the region  $W > 0$ , re-entering the  $W < 0$  domain at  $(u \approx 5, v \approx 0.62)$  and running back into the  $u$ -axis at  $u \approx 20.85$  (outside the plot). A similar fate occurs to the trajectory starting at  $(u = 0, v \approx 0.92)$  which re-enters the  $W < 0$  domain at  $(u \approx 11.1, v \approx 0.87)$  and finally collapses towards the  $u$ -axis at  $u \approx 46.4$ .

solution of (12). In this approach, the wave function is considered as a path integral  $\psi = \int \mathcal{D}g \exp(-I[g])$  over compact Euclidean ten-geometries having the argument of  $\psi$  (a nine-geometry or, here, just a point  $(u, v)$ ) as their boundary. The WKB-approximation scheme [17] then tells us to compute  $\psi \approx A \exp(-I)$  for points  $(u, v)$  near (or on) the zero-potential curve, where the Euclidean action  $I(u, v)$  is taken along the Euclidean trajectories.  $A$  is a prefactor that may be estimated by WKB-methods, once  $I$  is known. We would then have to take this  $\psi(W = 0)$  as a boundary condition for the Wheeler-DeWitt equation (12) and evolve it into the region  $W > 0$ . There,  $\psi$  is expected to develop a form  $\psi_{WKB} \approx B \cos(S) \equiv \frac{1}{2}B(\exp(iS) + \exp(-iS))$ . The phase  $S(u, v)$  is the action of a family of classical solutions that one may find using the Hamilton-Jacobi formalism [17]. The probability measure is provided by the prefactor  $B$ , and  $\psi$  can be viewed as describing a superposition of a family of classical universes (a particular one of course being ours). Some of these universes inflate in both scale factors, and some will eventually re-enter the quan-

tum region. Only two classical paths (namely, the ones emerging from the two instantons by smooth continuation) will undergo compactification of one scale factor – and although  $\psi$  can be expected to be highly peaked around these (as dominant contributions to the path integral), the situation is unsatisfactory as far as stability is concerned. (Remember that I refuse the introduction of a stabilizing supergravity-inspired matter coupling).

However, one encounters some complications in this model: To begin with, let us note that the Euclidean action  $I(u, v)$  is not unique, since there are several Euclidean trajectories connecting a given  $(u, v)$  with the origin. (Choosing the largest or the smallest of these would presumably not result into an approximate solution of the Wheeler-DeWitt equation at all, due to the effects at those points where two trajectories give equal  $I$ ).

Moreover, the interpretation of the region  $W < 0$  according to the standard methods is questionable: Since in a large portion of this region the curves  $W = \text{const}$  are timelike with respect to the Wheeler-DeWitt metric,  $\psi$  would be expected to be oscillatory there rather than exponential [6, 17]. This is however contrary to one’s intuition about what should happen when internal spaces approach the Planck scale.

## 2. Speculation

The question I would like to focus on is the following: What happens to the universe if it is described in the classical ( $W > 0$ ) region by a trajectory that approaches the zero-potential curve, enters the region of negative  $W$  and – when evolved further classically – recollapses? If  $\psi$  in the region  $W < 0$  (for  $u$  and  $v$  large) is indeed a semi-classical state, one should expect the universe to run into a singularity. There are some remarks about similar situations in the literature, stating either that the universe actually *will* recollapse classically [15] or that there might possibly occur some kind of collapse by tunneling [6]. In any case, such trajectories are usually not considered to be important in the description of our universe.

Now, let us proceed (very) heuristically and retain – against possible objections – the notion that a physically sensible state has to qualify the region  $W < 0$  as a “quantum” or a “classically forbidden” domain (at least if  $\ell_\Lambda \approx \ell_P$ ), irrespective of the fact that classical solutions do exist there formally. Then the question is: What happens if the universe re-enters into a classically forbidden region of the superspace? The intuitive answer is: tunneling. Since I have no better recipe at hand (maybe a path integral formalism would provide one), and since the Euclidean trajectories are in general considered as a viable tool describing tunneling, I try to *match a Euclidean trajectory smoothly to the*

*re-entering Lorentzian one.* This Euclidean trajectory will in general return to the curve  $W = 0$  and generate another semiclassical universe (which might recollapse again and so forth). Hence, we arrive at a procedure which tells us to evolve trajectories according to the classical equations of motion (11), but with the respective signs in each region. These (mixed) trajectories correspond to metrics undergoing an infinite succession of signature changes.

There are certainly many reasonable objections against such a procedure playing any role in the interpretation (or construction) of a state  $\psi$ . One such objection is provided by the fact that just matching trajectories smoothly as described above does not automatically lead to smooth ten-geometries displaying signature change (nor to a stationary point of the action  $S$  or  $I$ ). Optimal matching would rather demand vanishing extrinsic curvature [20] at  $W = 0$  (and this is in turn only possible for the two instantons, due to their turning point structure). But even this point is not so clear, because the recent discussion on signature change shows that weaker junction conditions may allow for reasonable classical evolution [21] too. I will return to this in the very last remark of my talk. However, even if the mixed trajectories do not represent admissible geometries, it may well be that they are linked with dominant contributions to the path integral with respect to some measure. (Recall that the structure of Euclidean trajectories in the minisuperspace of the model we consider is rather involved, and to find the dominant ones for a given point  $(u, v)$  is not a simple matter, even in the "standard" Hartle-Hawking approach [14]). Moreover, it is of course necessary to ask for the significance of the "time" variable  $t$  along the Euclidean pieces, and in which way such a construction is linked to physical predictions.

However, there might be a more accurate version of this procedure, where the matching of trajectories is not understood individually (one would expect a whole family of such mixed trajectories to build up a quantum state anyway). The weakest question one may make out of this is: Is there a solution of the Wheeler-DeWitt equation (12) which "drags" the universe approximately along such paths? This could happen either in terms of some oscillating WKB-type wave function (which is less likely) or in terms of a non-oscillating  $\psi$  displaying huge amplitudes in the according region of minisuperspace. (Recall that – except in the semiclassical WKB-approximation – there is no viable and commonly accepted interpretation of solutions to (12), especially when such high peaks occur in the absence of oscillations. This touches upon the most fundamental conceptual problems of quantum gravity [22], and therefore I cannot even give a precise statement which mathematical properties such a wave function should have.)

It is time now to look at the particular way the



Figure 3: A particular trajectory changing its character between Euclidean and Lorentzian (denoted "mixed trajectory" in the text) is displayed. It "oscillates" along the upper branch of the  $W = 0$  curve (dashed) and hence approaches asymptotically a constant value  $a_1 \rightarrow \ell_\Lambda \sqrt{3}$ . The asymptotic boundary condition near the origin (from where the numerical evolution starts) has been chosen as  $u^{-5/2}v \rightarrow 0.25$ .

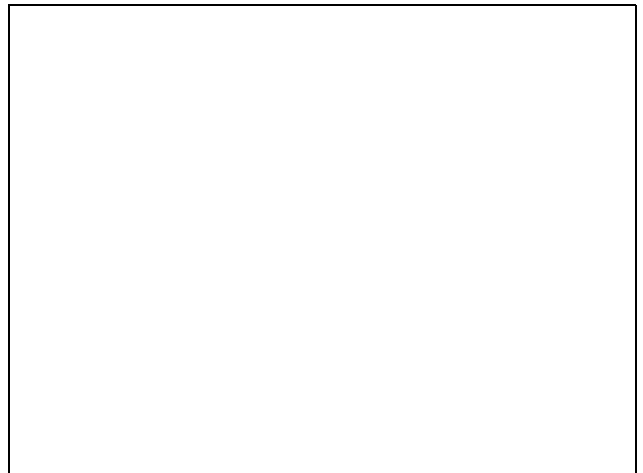


Figure 4: This plot shows another mixed trajectory, now "oscillating" around the lower branch of the zero-potential curve. The scale factor belonging to  $\mathbf{S}^6$  approaches  $a_2 \rightarrow \ell_\Lambda \sqrt{15}$ . The asymptotic behaviour near the origin is  $u^{-1/2}v \rightarrow 0.5$ .

Euclidean/Lorentzian mixed trajectories behave in the model we are considering. From simple numerical techniques it follows that there is a preferred region on the curve  $W = 0$  from which such trajectories emerge; this region seems to consist of two pieces which are located near the turning points of the two instantons, and of points such that two different Euclidean trajectories starting from the origin and meeting there have approximately the same action  $I$ .

Figs.3 and 4 show two examples of mixed trajectories. They undergo "oscillations", thereby wandering

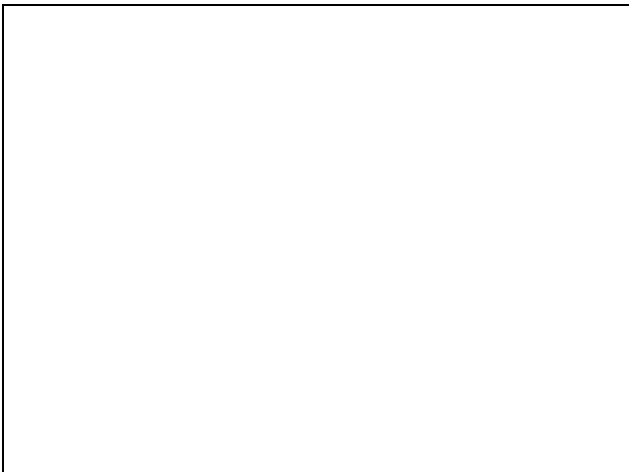


Figure 5: The graphs of the functions  $a_1(t)$  and  $a_2(t)$  for the trajectory displayed in Fig.4 are shown. The gauge condition defining  $t$  is  $N = 1$ . In order to get definite units, we have chosen  $\Lambda = 1$ , hence  $\ell_\Lambda = 1$ .



Figure 6: This provides a magnification of Fig.5 in order to show how  $a_2(t)$  performs "damped" oscillations arounds its limiting value.

along one of the two asymptotic branches of  $W = 0$ . One of the two scale factors thereby approaches a finite limiting value in each case, the other one blows up. In Fig.5, the data from Fig.4 are re-expressed in terms of  $(a_1(t), a_2(t))$ , where the time coordinate  $t$  refers to the gauge  $N = 1$ . Fig.6 provides a magnification of the same plot, showing that the "oscillation" of  $a_2$  is actually damped. Hence mixed trajectories correspond to metrics which "oscillate" in signature and display – as far as the values of the scale factors are concerned – perfect Kaluza-Klein behaviour.

### 3. Outlook

So, can the universe "reappear" due to quantum tunneling? It is a subject for further research to look for solutions of (12) describing such a behaviour. Let

me however conclude with some remarks. First, such a compactification and stabilization mechanism would be quite general and independent of most features of matter coupling. It just requires the internal space to have non-vanishing Ricci curvature. (Note that in the  $\mathbf{S}^3 \times \mathbf{T}^6$  model compactification of the  $\mathbf{T}^6$  cannot happen in this way because the lower asymptotic branch of  $W = 0$  is missing). Moreover, the compactified scale factor approaches  $\approx \ell_\Lambda$ , hence the whole mechanism works only if  $\Lambda$  does not decay. It must be a fundamental part of the action (and not just mimicking a  $V(\phi)$  which would disappear after an inflationary phase [15]). A possible success of computations along these lines could certainly be connected with a new aspect in the interpretational problems of the state  $\psi$ , because in some sense the universe would remain permanently a quantum one. One would have to work out how the arrow of time comes about, and how the universe as a whole is experienced. Naively, one would expect that a state in which  $a_2$  compactifies (cf. Fig.5) describes an effectively three-dimensional world expanding as  $a_1(t)$ . But why do we observe a classical space-time with Lorentzian signature, and how does  $a_1$  evolve with respect to an (effectively) Lorentzian time?

Let me add a comment that emerged in discussions at this meeting: When trying to compute  $I(u, v)$  along Euclidean trajectories, one would – as already mentioned – encounter different, competing contributions  $I_1(u, v)$  and  $I_2(u, v)$ , say. This degree of non-uniqueness can – by virtue of rather involved branching phenomena – result in the mathematical structure necessary to develop non-expected compactification solutions and at the same time into the feature of being "non-integrable" in some sense. Hence, the difficulties in finding analytic solutions may possibly be compensated by new physical effects. In this concern, there is also still something to learn about the  $k = 0$  model (where the  $\mathbf{S}^3$  would compactify along the lines described above).

My last remark concerns the form of the Wheeler-DeWitt equation (12). One might consider the Euclidean solutions as physical (i.e. classical) ones (and not just tools to integrate (12) or to evaluate a path integral). In other words, a physical signature change would actually be a thing to happen. In this case, it is worth noting that the mixed trajectories extremize the action  $S_{\text{mod}}$ , obtained from (7) after replacing  $W$  by its absolute value  $|W|$ . To what extent they represent physical solutions is not entirely clear, because the question of which junction conditions should apply at the surface of signature change is still a bit open [20, 21]. In any case, the "classical signature change" approach favours weaker conditions (continuity of the extrinsic curvature instead of vanishing), and the mixed trajectories I described are likely to be accepted as classically sensible solutions. Clearly, such

an approach alters the standard foundations of quantum gravity. Following the spirit of my speculations, one would modify the Wheeler-DeWitt equation accordingly, and thus arrive at

$$\partial_{uv}\psi(u, v) = -\Lambda_{\text{eff}}^{-8} |W(u, v)| \psi(u, v) \quad (13)$$

instead of (12). Expressed briefly, treating signature change as a classical effect, the substitution  $p_u \rightarrow -i\partial_u$ ,  $p_v \rightarrow -i\partial_v$  in the Euclidean Hamiltonian  $\mathcal{H}_E = N(-p_u p_v - \Lambda_{\text{eff}}^{-8} W)$  leads to an equation like (12) but with  $W$  replaced by  $-W$ . In contrast, the usual quantum cosmology approach amounts to perform the Euclideanized substitution  $p_u \rightarrow \partial_u$ ,  $p_v \rightarrow \partial_v$  in  $\mathcal{H}_E$ , which leads back to (12). The statement that the use of either Lorentzian or Euclidean geometries makes no difference in deriving the Wheeler-DeWitt equation [15] is only true if the latter are of no classical significance. Postulating Euclidean signature in the "quantum domain" of minisuperspace (which may well be a "classical" domain in this context, especially if  $\ell_\Lambda \gg \ell_P$ ), leaves the zero-potential curve as the set of points where signature change *may* happen. Assuming further that it *does* happen there, one arrives at (13).

A modified Wheeler-DeWitt equation as in (13) is exactly what has been proposed recently by Martin [23]. In his paper, Martin also shows (in a FRW plus scalar field minisuperspace model) that the modified Wheeler-DeWitt equation allows less physically sensible boundary conditions than the standard one. If such an approach turns out to be fruitful for quantum cosmology, the speculations made here would turn out to be much less speculative than in the usual context, but would simply be a heuristic description of a particular candidate semiclassical state  $\psi \sim \Sigma \cos(S_{\text{mod}})$  or  $\Sigma \exp(iS_{\text{mod}})$ . Maybe the following (very last) speculation is true: states built around mixed trajectories appear in the standard cosmological approach as highly peaked in the  $W \approx 0$  region of superspace, whereas in the classical signature change picture they emerge as oscillating wave functions.

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