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QUANTUM EVOLUTION OF THE UNIVERSE: A GAUGELESS APPROACH

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We discuss the quantum evolution of the Universe from inflation till the classical Friedmann stage in a gaugeless approach to the Einstein gravity . In the gaugeless method the physical phase space of the Einstein gravity with the Dirac– Arnowitt –Deser–Misner metric is constructed without using a gauge fixing condition by explicitly resolving the constraints for nonphysical fields. From the reduced Einstein action we get a new energy momentum tensor for independent dynamical fields. The corresponding energy and time are in direct analogy with the "spectral energy" and "observable time" for a relativistic particle.

1. Introduction

The problem of quantization of gravity from the beginnings until recently encountered the difficulty of defining the physical degrees of freedom of the gravitational field [1–10]. The procedure of identification of physical variables and their separation from nonphysical ones has been called the reduction procedure. There are two ways to realize the reduction in the classical and quantum theories: the gaugeless and the gauge fixing ones. In the former, independent physical variables are constructed by explicitly resolving the constraints. To avoid the difficulties with the resolution of very complex constraints in gravity one commonly uses the gauge fixing method based on introducing into the theory some new "gauge constraints" [1, 9]. However, such coordinate fixation due to the nonlinear character of gravitation encounters the problem of determination of the class of "admissible gauges" which allows one to obtain gauge-independent results [11]. Recall that the gauge equivalence theorem has been proved only for asymptotically flat space-times [7]. It seems to us that the problem of definition of admissible gauges is not easier than that of resolving the constraints.

In the present paper, we try to follow the gaugeless reduction [12–16] of gravity, based on an explicit resolution of the classical Hamiltonian constraints for nonphysical field momenta and the corresponding fields coordinates .

The application of the gaugeless approach to the relativistic particle model is quite simple. Resolution of the mass-shell constraint for a relativistic particle

$$\mathcal{H} = \frac{1}{2}(-p_0^2 + p_i^2 + m^2) = 0 \,, \tag{1.1}$$

leads to the notion of particle energy

$$p_0 = \pm\omega \;; \quad \omega = \sqrt{p^2 + m^2} \,, \tag{1.2}$$

and resolution of the equation of motion of the corresponding coordinate gives us the definition of the observable time. It is very attractive to transfer these clear notions of energy and observable time for a relativistic particle to the case of gravity. We shall deal with this analogy and show that the resolution of constraints and the corresponding equations of gravity lead to the new notions of "spectral energy" $\mathcal{E}_s$ of the type of (1.2) and "spectral time" T_s as a variable canonically conjugated to this energy.

The main goal of our paper is to clear up the physical meaning of spectral energy and spectral time in gravity in the context of quantum cosmology. The determination of the physical time is a key problem in classical and quantum gravity. There is a lot of speculations on this subject, here we try to continue these attemps in the spirit of the "internal Schrödinger interpretation" (see e.g. [17]). In our approach we reformulate the theory in terms of invariant variables.

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That allows us to define time as a global invariant, associated with the reduced action.

2. Gaugeless reduction of the Einstein gravity

2.1. The Hamiltonian form

We start with the conventional scalar curvature action including the electromagnetic field to control the reduction procedure

$$W[g,A] = -\int d^4X\sqrt{-g}\left(\frac{1}{2\kappa^2}{}^{(4)}R(g) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right). \tag{2.1}$$

It is well known that the Einstein equations

$$\frac{\delta W}{\delta g_{0\mu}} = 0$$

are Lagrange constraints. In the Hamiltonian approach they correspond to secondary constraints and the reduction consists in their explicit resolution with respect to certain momentum and coordinate.

The Hamiltonian approach with the instant form of the dynamics enforces us to assume that the space time manifold $\mathcal{M}$ can be represented as $\mathcal{M} = \mathcal{R}\times\,\Sigma$, where Σ is a three dimensional surface. The space-time foliation is realized by introducing the so called embedding variables $X(\mathbf{x},t)$ [18] which are maps from a point $\mathbf{x}$ of the surface Σ to a space-time point X of the manifold $\mathcal{M}$, and t labels the leaves of the foliation. This foliation leads to the well known Dirac–Arnowitt–Deser–Misner (Dirac–ADM) metric [1, 4]

$$ds^2 \;=\; N^2(dt)^2 - a^2h_{ik}(dx^i + N^idx^0)(dx^k + N^kdx^0) \tag{2.2}$$

where N is the lapse function, N^i is the shift vector , a is the "scale–space" component of metric, h_{ik} is the "graviton component" with the determinant equal to unity:

$$\sqrt{-g} = Na^3\ , \quad \det(h_{ik}) = 1\ , \quad a = \exp\mu\ . \tag{2.3}$$

The Einstein –Hilbert action (2.1) in terms of this metric possesses a manifest symmetry under the following group of transformations [19]

$$\begin{aligned} t \;&\rightarrow\; t' = t'(t)\\ x^i \;&\rightarrow\; x^{i'} = x^{i'}(t, x^1, x^2, x^3). \end{aligned} \tag{2.4}$$

Let us rewrite the action (2.1) in terms of the embeddings. The scalar curvature can be decomposed into three terms: the "kinetic" one, $\mathcal{K}$, the three-dimensional curvature ${}^{(3)}R$, and the "surface" one, Σ:

$$\begin{aligned}
{}^{(4)}R \;&=\; -\mathcal{K} \;+\; {}^{(3)}R + 2\Sigma, && (2.5)\\
\mathcal{K} \;&=\; -6\frac{\overset{\odot}{\mu}{}^2}{N^2} \;+\; \frac{\overset{\odot}{h}{}^2}{4N^2}, && (2.6)\\
{}^{(3)}R \;&=\; \frac{4}{a^2}\left[h^{kl}\nabla_l\partial_k\mu + \frac{1}{2}\partial_k\mu\partial^k\mu\right] + \frac{1}{a^2}R(h), && (2.7)\\
\Sigma \;&=\; \frac{1}{Na^3}\left[\partial_k\left(a\partial^kN\right) - 3\breve{\partial}_0\left(\frac{a^3\,\overset{\odot}{\mu}}{N}\right)\right], && (2.8)
\end{aligned}$$

where we introduce the notations

$$\begin{aligned}
\breve{\partial}_0\phi \;&=\; \partial_0\phi \;-\; \partial_k(\phi N^k) && (2.9)\\
\overset{\odot}{\mu} \;&=\; \dot{\mu} - \frac{1}{3a^3}\partial_k(a^3N^k) \equiv \frac{1}{3a^3}\breve{\partial}_0(a^3), && (2.10)\\
\overset{\odot}{h}{}^i_k \;&=\; h^{il}\left(\dot{h}_{kl} - \nabla_lN_k - \nabla_kN_l + \frac{2}{3}h_{kl}\partial_iN^i\right) && (2.11)\\
R(h) \;&=\; \frac{1}{4}\partial_ih^k{}_l(\partial^ih^l{}_k - 2\partial^lh^i{}_k) \;+\; \partial_k\partial_lh^{kl}. && (2.12)
\end{aligned}$$

The canonical momenta conjugated to μ, h, and A are

$$\begin{aligned}
P_{(\mu)} \;&=\; \frac{\partial\mathcal{L}}{\partial\dot{\mu}} = -\frac{6a^3}{N\kappa^2}\overset{\odot}{\mu}, && (2.13)\\
P^k_{(h)l} \;&=\; \frac{\partial\mathcal{L}}{\partial\dot{h}^l_k} = \frac{a^3}{4N\kappa^2}\,\overset{\odot}{h}{}^k_l, && (2.14)\\
P_{(A)k} \;&=\; \frac{\partial\mathcal{L}}{\partial\dot{A}^k} = \frac{a}{N}\left(\dot{A}_k - \partial_kA_0 - N^lF_{lk}\right) = \frac{a}{N}\overset{\odot}{A}_k. && (2.15)
\end{aligned}$$

Here $\partial_jh^k{}_l \;=\; h^{ki}\partial_jh_{il}$, $N_l \;=\; h_{li}N^i$ and ∇_l is a covariant derivative in metric h_{ik},

In terms of the variables

$$\Phi = (h, A), \qquad P_{(\Phi)} = (P_{(h)}, P_{(A)}), \qquad \mu, \quad P_{(\mu)}$$

the action (2.1) has the form

$$W \;=\; \int d^3x\,dt\left[\sum_{\Phi=(h,A)} P_\Phi\,\overset{\odot}{\Phi} + P_{(\mu)}\overset{\odot}{\mu} - N\mathcal{H}_E - S_\Sigma\right]. \tag{2.16}$$

Here we keep the surface term (2.8), taking into account (2.13):

$$S_\Sigma = Na^3\frac{\Sigma}{\kappa^2} = \breve{\partial}_0\frac{P_{(\mu)}}{2} + \frac{\partial_k(a\partial^kN)}{\kappa^2}, \tag{2.17}$$

in contrast to the conventional ADM approach where this term is omitted; $\mathcal{H}_E$ is the Einstein energy density

$$\mathcal{H}_E = a^3\left[-\frac{\kappa^2}{2\cdot6}\frac{P^2_{(\mu)}}{a^6} + T^0{}_0(h,A)\right]\ ; \tag{2.18}$$

where ${T^0}_0(h,A) \equiv {T^0}_0(h) + {T^0}_0(A)$ is the zero-zero component of the energy momentum tensor

$$\begin{aligned} {T^0}_0(h) &= \frac{4\kappa^2 P^2_{(h)}}{2a^6} + \frac{{}^{(3)}R}{2\kappa^2 a^2}; \\ {T^0}_0(A) &= \frac{1}{2a^4}\left(E_iE^i + \frac{F_{ij}F^{ij}}{2}\right). \qquad (2.19) \end{aligned}$$

The action (2.16) can be represented in a more detailed form as

$$\begin{aligned} W = \int d^3x\, dt \Bigg[\sum_{\Phi=(h,A)} P_{(\Phi)}\dot{\Phi} + P_{(\mu)}\dot{\mu} + A_0\partial_k E^k \\ - N\mathcal{H} + N^k\mathcal{P}_k - \frac{\dot{P}_{(\mu)}}{2} - \partial_k S^k \Bigg]. \qquad (2.20) \end{aligned}$$

In Eq.(2.20) the surface term is

$$S^k = -\frac{a}{\kappa^2}\partial^k N + 2P^k_{(h)l}N^l - \frac{1}{6}N^k P_{(\mu)} + A_0E^k. \qquad (2.21)$$

and A_0, N, N^k are concidered to be Lagrange factors for the constraints

$$\mathcal{H} = 0\,, \quad \mathcal{P}_k = 0\,, \quad \partial_k P^k_{(A)} = 0\,, \qquad (2.22)$$

where $\mathcal{P}_k$ is the Einstein momentum density:

$$\begin{aligned} \mathcal{P}_k &= \frac{a^3}{3}\partial_k\left(\frac{P_{(\mu)}}{a^3}\right) + Na^3{T^0}_k(h,A), \qquad (2.23) \\ {T^0}_k(h,A) &\equiv {T^0}_k(h) + {T^0}_k(A), \qquad (2.24) \\ {T^0}_k(h) &= \frac{1}{Na^3}\left[2\partial_l P^l_{(h)k} - \partial_k h^i_l P^l_{(h)i}\right]; \\ {T^0}_k(A) &= \frac{1}{Na^3}F_{kl}E^l. \qquad (2.25) \end{aligned}$$

To complete, we should like to note that the expression

$$\begin{aligned} {T^k}_k(h,A) &\equiv {T^k}_k(h) + {T^k}_k(A), \qquad (2.26) \\ {T^k}_k(h) &= -3\frac{4\kappa^2 P^2_{(h)}}{2a^6} + \frac{{}^{(3)}R}{2\kappa^2 a^2} : \\ {T^k}_k(A) &= -{T^0}_0(A) \qquad (2.27) \end{aligned}$$

is the spatial trace of the total energy-momentum tensor for gravitons (h) and photons (A) in the Einstein equation $\delta W/\delta\mu = 0$. With the constraints (2.22) this equation has the form (see also [16])

$$\begin{aligned} S_\Sigma &= Na^3\left[{T^0}_0(h,A) + \frac{\operatorname{tr} T(h,A)}{2}\right]; \\ &(\operatorname{tr} T = {T^0}_0 + {T^k}_k). \qquad (2.28) \end{aligned}$$

The action (2.20) describes the generalized Hamiltonian dynamics for (μ, h_{kl}, A_k) and $P_{(\mu)}, P_{(h)}, P_{(A)}$ with the constraints (2.22).

2.2. Reduction of the phase space

To attain the reduced phase space we shall act by direct analogy to the relativistic particle case and QED. In these cases the resolution of constraints leads to the construction of gauge invariant variables (QED) [13, 15] and to the observable time as a global invariant of the reparametrization group (a relativistic particle) [16]. The same gauge invariant variables for gravity have been constructed in the framework of the cosmological perturbation theory, with the choice of the conformal time [20]. Here we discuss the dynamical aspect of this gaugeless reduction connected with the construction of the "spectral Hamiltonian" and "spectral time", by the resolution of the "energy" constraint $\mathcal{H}_E = 0$ with respect to the space-scale momentum $P_{(\mu)}$ by analogy with the "instant form" of relativistic particle dynamics. We should like to note that there is a possibility to choose another form of dynamics which corresponds to resolution of the energy constraint with respect to a different momentum. Moreover, these forms can be nonequivalent.

The explicit resolution of the constraint $\mathcal{H}_E = 0$ allows us to represent $P_{(\mu)}$ as a functional of the physical variables Φ, $P_{(\Phi)}$, μ. This constraint has two solutions

$$P_{(\mu)} = \mp F; \qquad F = \frac{\sqrt{12}}{\kappa}a^3[{T^0}_0(h,A)]^{\frac{1}{2}} \qquad (2.29)$$

The radicand in Eq.(2.29) is not positive definite. In the classical case the positivity condition for ${T^0}_0(h,A)$ restricts the admissible regions for the physical variables h, A and their momenta, whereas in quantum theory negative values of ${T^0}_0(h,A)$ can lead to physical phenomena like the tunnel effect.

According to the reduction procedure, let us substitute the solution (2.29) into the initial action (2.16) and consider the two terms ${}_{(\mu)}\overset{\odot}{\mu} \;\; - S_\Sigma$. On the constraints-shell (2.29) the total derivative in S_Σ (2.17) can be represented as

$$\begin{aligned} \breve{\partial}_0 F \equiv \frac{\partial F}{\partial\mu}\breve{\partial}_0\mu + \frac{\partial F}{\partial(\partial_i\mu)}\breve{\partial}_0\partial_i\mu + \frac{\partial F}{\partial(\Delta\mu)}\breve{\partial}_0\Delta\mu + \\ + \sum_\Phi\left[\frac{\partial F}{\partial\Phi}\breve{\partial}_0\Phi + \frac{\partial F}{\partial(\partial_i\Phi)}\breve{\partial}_0(\partial_i\Phi) + \frac{\partial F}{\partial P_{(\Phi)}}\breve{\partial}_0(\partial P_{(\Phi)})\right]. \\ (2.30) \end{aligned}$$

One can see that (2.30) in the action (2.16) represents a sum of the equations of motion for the fields $\mu, \Phi, P_{(\Phi)}$:

$$\begin{aligned} S_\Sigma \equiv \underline{N}a^3\left[{T^0}_0(h,A) + \frac{\operatorname{tr} T(h,A)}{2}\right] + \\ + \frac{1}{2}\sum_\Phi\left[\frac{\partial F}{\partial\Phi}\breve{\partial}_0\Phi + \frac{\partial F}{\partial(\partial_i\Phi)}\breve{\partial}_0(\partial_i\Phi) + \frac{\partial F}{\partial P_{(\Phi)}}\breve{\partial}_0(P_{(\Phi)})\right], \\ (2.31) \end{aligned}$$

just as the identity $\dot{\omega} \equiv \sum \dot{p}_i \frac{\partial W}{\partial p_i}$ represents a sum of the equations of motion for a relativistic particle. $\underline{N}$ denotes the lapse function N on the constraint-shell $\mathcal{H}_E = 0$:

$$\underline{N} =\overset{\odot}{\mu} \frac{6a^3}{\kappa^2 F} \equiv\overset{\odot}{\mu} \frac{\sqrt{3}}{\kappa\sqrt{{T^0}_0}} \tag{2.32}$$

The identity (2.31) means that the classical dynamics of the metric (2.28), (2.29), (2.13) defines the dynamics of the "matter" field (in our case gravitons and photons). This fact was discovered by V. Fock [21] and then rediscovered by a number of authors.

By using Eqs. (2.31) and (2.32) in the form

$$F \overset{\odot}{\mu}= \underline{N}\frac{\kappa^2 F^2}{6a^3} = \underline{N}a^3 \cdot 2{T^0}_0(h, A) \tag{2.33}$$

we can easily get the result for the reduced action (2.16) on the constraint-shell $\mathcal{H}_E = 0$

$$\begin{aligned} W^{\mathrm{Red}}_{\pm} &= \int d^3x\, dt \sum_{\Phi} P_{(\Phi)} \overset{\odot}{\Phi} \mp \frac{1}{2}\frac{\partial F}{\partial \Phi}\breve{\partial}_0\Phi \\ &\mp \frac{1}{2}\frac{\partial F}{\partial(\partial_i\Phi)}\breve{\partial}_0(\partial_i\Phi) \mp \frac{1}{2}\frac{\partial F}{\partial P_{(\Phi)}}\breve{\partial}_0(P_{(\Phi)}) \mp \underline{N}\mathcal{H}_{\mathcal{P}} \end{aligned} \tag{2.34}$$

where

$$\begin{aligned} \mathcal{H}_{\mathcal{P}} &= a^3\left({T^0}_0(h, A) - \frac{\mathrm{tr}\, T(h, A)}{2}\right) \\ &\equiv a^3\left[{T^0}_0(A) + \frac{4P^2_{(h)}\kappa^2}{a^6}\right]. \end{aligned} \tag{2.35}$$

Note that the lapse function (2.32) and the Hamiltnian (2.35) are similar to the Poincare [22] mass-shell energy and lapse function for a relativistic particle:

$$\mathcal{H}_{\mathcal{P}} = (p_i^2 + m^2); \qquad \underline{N} = \frac{\dot{x}_0}{\sqrt{p^2 + m^2}}. \tag{2.36}$$

We can see that the reduced action on the solution of classical equation leads to the generalization of the Tolman energy momentum tensor [23] for " matter " field (M) including gravitons [16]

$$T_{(\mathrm{Tolman})}{}^{\mu}_{\nu}(h, M) = T^{\mu}_{\nu}(h, M) - \delta^{\mu}_{\nu}\frac{\mathrm{tr}\, T(h, M)}{2}. \tag{2.37}$$

Expressions (2.34), (2.35), (2.37) can be regarded as a basis for the construction of a Hamiltonian scheme in terms of gauge-invariant variables.

3. Spectral time

The time problem is many-sided [17]. We should like to discuss a part of this problem concerning the observability. In gravity all observables, including time, should be invariant with respect to the group of transformations (2.4). In relativistic mechanics there are two "times" that are invariant under time reparametrization:

(i) a local quantity — the proper "time" interval (in cosmology it is the Friedmann time)

$$dT_F = \underline{N}dt; \tag{3.1}$$

(ii) the spectral time as a global quantity conjugated to the spectral energy.

Note that the proper time, in the case of a relativistic particle, coincides with the spectral time only for a particle at rest. For a moving particle, according to Eq.(2.32), we can see that the proper (Friedmann) time and the spectral one are connected by the Lorentz transformation:

$$T_F = \frac{m}{\omega}x_0\,. \tag{3.2}$$

The question arises: what is the analog of this observable time x_0 in gravity ? As has been shown in the cosmological perturbation theory [23], the explicit resolution of the constraints (2.22) generally allows one to express $P_{(\mu)}$ and μ as functionals of the variables $\Phi = (A, h)$ and $P_{(\Phi)} = (E, P_h)$ within the zero mode sector (compare to Eq.(2.10))

$$\mu(t) = \mu_0(t) + \mu_L[\Phi, P_{(\Phi)}].$$

This allows us to pass to the time variable μ_0 defining $dt\underline{N} = d\mu_0 N_0$. To define the observable time, let us suppose that we know the spectrum of the Poincare Hamiltonian (2.35)

$$\begin{aligned} H_{\mathcal{P}}[\mathcal{P}_\phi, \phi, \mu_0] &= \int d^3x N_0 \mathcal{H}_{\mathcal{P}}, \\ H_{\mathcal{P}}[\hat{\mathcal{P}}_\phi, \hat{\phi}, \mu_0]\psi_\varepsilon &= \varepsilon_s(\mu_0)\psi_\varepsilon(\phi); \\ i[\hat{\mathcal{P}}_\phi(x), \hat{\phi}(y)] &= \delta^3(x - y) \end{aligned} \tag{3.3}$$

as a function of μ_0. Then one can write down the spectral decomposition for the amplitude of probability to find our system in the configuration $\phi(T)$ at the "time" T if at the beginning of "time" it was in the configuration $\phi(0)$:

$$\begin{aligned} &G(\phi(T), \phi(0)|T_s) \\ &= \sum_s A^{(+)} \exp\left\{-i\int_{\mu_0(0)}^{\mu_0(T)} d\mu_0 \varepsilon_s(\mu_0)\right\} \\ &\qquad\times \psi_{\varepsilon_{(+)}}(\phi(T))\psi^*_{\varepsilon_{(+)}}(\phi(0)) \\ &+ A^{(-)} \exp\left\{+i\int_{\mu_0(0)}^{\mu_0(T)} d\mu_0 \varepsilon_s(\mu_0)\right\} \\ &\qquad\times \psi_{\varepsilon_{(-)}}(\phi(T))\psi^*_{\varepsilon_{(-)}}(\phi(0)), \end{aligned} \tag{3.4}$$

where $A^{(\pm)}$ are coefficients of decomposition over two different solutions of the energy constraint $\mathcal{H}_E = 0$.

From this spectral decomposition we can define the "spectral time "

$$\int_{\mu_0(0)}^{\mu_0(T)} d\mu_0 \varepsilon_s(\mu_0) = \mathcal{E}_s\left[T_s(\mu_0(T)) - T_s(\mu_0(0))\right] \quad (3.5)$$

within the observable conserved energy $\mathcal{E}_s$. The energy $\mathcal{E}_s$ can be defined from the principle of correspondence with the classical theory or with other known physical limits. In the simplest case of stationary gravity fields and matter (when $H_{\mathcal{P}}$ represents a C-number) $\mathcal{E}_s$ has been calculated by Tolman [26] and it coincides with the total mass: $\mathcal{E}_s = M$. The spectral time in this case is the proper one, like that for a relativistic particle at rest.

4. Spectral History of the Universe

Let us repeat the above scheme for the case of a homogeneous space-time with the metric

$$\begin{aligned}(ds)^2 \; &= \; N^2(t)\, dt^2 \\ &- \; a^2(t)\left(1 - \frac{kr^2}{4r_0^2}\right)^{-2} h_{ij}(t)\, dx^i\, dx^j, \end{aligned} \quad (4.1)$$

where $k = 0, +1, -1$ corresponds to the flat, close, and open three-dimensional space, r_0 being the parameter of curvature. The Einstein action for this metric has the form

$$W = V_{(3)} \int_0^T dt \left[P_{(h)}\dot{h} + P_{(\mu)}\dot{\mu} - N\mathcal{H}_E - \frac{\dot{P}_{(\mu)}}{2}\right] \quad (4.2)$$

where $V_{(3)}$ is the three-dimensional volume,

$$\begin{aligned} \mathcal{H}_E \; &= \; a^3\left[-\frac{\kappa^2 P_{(\mu)}^2}{2\cdot 6a^6} + T_0^0\right]; \\ T_0^0 \; &= \; T_0^0(h) + T_0^0(M), \quad (4.3) \\ T_0^0(h) \; &= \; \frac{4\kappa^2 P_{(h)}^2}{2a^6} - \frac{3k}{\kappa^2 r_0^2 a^2}, \quad (4.4) \end{aligned}$$

$T_0^0(M)$ is the energy density of "matter", which is considered to be the nondynamic content of the Universe, for example, it can include radiation (ε_R), dust (ε_D) and a λ term:

$$T_0^0(M) = \frac{\varepsilon_R}{a^4} + \frac{\varepsilon_D}{a^3} + \lambda,$$

$$T_k^k(M) = a^{-3}\frac{\partial}{\partial\mu}(a^3 T_0^0) = -\frac{\varepsilon_R}{a^4} + 3\lambda. \quad (4.5)$$

The energy constraint $\mathcal{H}_E = 0$ has two solutions:

$$P_{(\mu)} = \mp F\,; \quad F = \frac{\sqrt{6}}{\kappa} a^3\sqrt{2T_0^0} \quad (4.6)$$

and the corresponding reduced actions are

$$W_{\pm}^{\rm Red} \; = \; V_{(3)}\int_0^T dt\left[P_{(h)}\dot{h} \mp \underline{N}\mathcal{H}_P \pm \frac{1}{2}\dot{P}_{(\mu)}\frac{\partial F}{\partial P_{(\mu)}}\right] \quad (4.7)$$

where $\mathcal{H}_P$ is the Poincare Hamiltonian (2.35)

$$\mathcal{H}_P = a^3\left[\frac{4\kappa^2 P_{(h)}^2}{a^6} + \frac{\varepsilon_R}{a^4} + \frac{1}{2}\frac{\varepsilon_D}{a^3} - \lambda\right], \quad (4.8)$$

which does not contain the curvature ${}^{(3)}R$, and

$$\underline{N} = \frac{\sqrt{3}\dot{\mu}}{\kappa[T_0^0]^{1/2}} \quad (4.9)$$

is the lapse function on the constraint $\mathcal{H}_E = 0$. We would like to emphasize that our approach differs from the conventional one by the following :

(i) all surface terms in the initial Einstein theory $({}^{(4)}R)$ are taken into account;

(ii) instead of the Friedmann time

$$T_F = \int_0^T N dt = \int_{a(0)}^{a(T)} \frac{da\,\sqrt{3}}{a\,\kappa[T_0^0]^{1/2}} \quad (4.10)$$

we use the spectral time T_s

$$T_s = \frac{V_{(3)}}{\mathcal{E}_s}\int_{a(0)}^{a(T)} dT_F(a)\mathcal{H}_{\mathcal{P}}(a)$$

as the observable one.

This difference leads to new cosmological consequences for the Universe at every stage of evolution: the Misner one, radiation, dust and de Sitter ones. Let as compare the Friedmann and spectral laws on each stage.

4.1. The Misner anisotropic Universe

The main difference of the spectral time from the Friedmann one is a monotonic dependence on the scale factor a. The limit of small a corresponds to small spectral time in contrast to the Friedmann case. For vanishing a the first term in the Poincare Hamiltonian dominates and the reduced action describes the Misner anisotropic Universe (4.7)

$$\begin{aligned} W^{Red} \; &= \; V_{(3)}\int_{\mu(0)}^{\mu(T)} d\mu\Big[P_{(h)}\frac{\partial}{\partial\mu}h \\ &\mp 2\sqrt{6P_{(h)}^2} \pm \frac{\partial}{\partial\mu}\sqrt{6P_{(h)}^2}\Big], \end{aligned} \quad (4.11)$$

which is independent of the coupling constant κ. The classical dynamics is described by the conservation of momentum $\frac{\partial}{\partial\mu}P_{(h)} = 0$, so that the last term does not contribute.

The spectral time at this stage coincides with the logarithm of the space scale, while the Friedmann law (4.10) is $T_F = a^3/\kappa^2\sqrt{6P_{(h)}^2}$.

In quantum theory, in the spectral decomposition of the Green function (3.4) the spectral time has an absolute beginning for an observer (who writes this function in his note-book).

The positive sign of time corresponds to the expansion of the Universe and the negative sign to the contraction of the (anti-)Universe.

The "observer" sees a small Universe be created with a finite volume and density and undergo inflation with respect to the "spectral time". In other words, the anisotropic stage protects the "observer" in the expanding Universe from the initial Hawking singularity.

The process of inflation changes the matter content of the Universe due to its creation. We leave the description of this creation for forthcoming papers and consider here the next stage: radiation.

4.2. Radiation stage

At the radiation stage $(P_{(h)} = \varepsilon_D = \lambda = 0)$, the reduced action has the form

$$\begin{aligned} W^{\rm Red}_{\pm}(R) &= \mp V_{(3)} \int_0^T dt\, \frac{N}{a}(\varepsilon_R) \\ &= \mp [V_{(3)}\varepsilon_R][\eta(T) - \eta(0)] \end{aligned} \qquad (4.12)$$

where η is the conformal time which plays the role of the spectral one in the spectral decomposition of the wave function

$$\psi_R = A^{(+)} e^{iW^{\rm Red}_{(+)}(R)} + A^{(-)} e^{iW^{\rm Red}_{(-)}(R)}. \qquad (4.13)$$

The conformal expansion law is monotonic and goes to infinity for all types of space, hyperbolic $(k = -1)$, flat $(k = 0)$, and closed $(k = +1)$, in contrast to the Friedmann law described for the case $(k = +1)$ by the function $a(T_F)$, which is given on a closed circle (for more details see Refs. [16]) and violates the causality.

4.3. Dust stage

At the dust stage $(P_{(h)} = \mathcal{E}_d = \lambda = 0)$ the reduced action becomes

$$\begin{aligned} W^{\rm Red}_{\pm}(D) &= \mp V_{(3)} \int_0^T dt \underline{N} \frac{\mathcal{E}_d}{2} \\ &= \mp \frac{V_3 \mathcal{E}_d}{2} \left(T_F(T) - T_F(0)\right). \end{aligned} \qquad (4.14)$$

The spectral time coincides with the Friedmann one for any type of the Universe $k = 0, \pm 1$ (like the case of a relativistic particle at rest) with the decomposition of a wave function of the type of (7.13). The spectral energy is only half the mass of the Universe due to the homogeneous approximation, in accordance with Tolman's result of 1930 [26].

4.4. The de Sitter stage and the zero energy expansion

To complete, let us compare here also the spectral expansion law with the Friedmann one for the de Sitter stage $P_{(h)} = \mathcal{E}_R = \mathcal{E}_D = 0$ in the space with $(k = 0)$. The reduced action in this case has the form

$$W_{(\pm)} = \pm (V_{(3)}\lambda) \int_0^T dt \underline{N} a^3 = \pm (V_{(3)}\lambda^{\frac{1}{2}}) \frac{a^3}{\sqrt{3}}\, \kappa. \qquad (4.15)$$

The role of spectral time is played by the space volume (a^3) instead of $\log a$ for the Friedmann time. From the point of view of our observer the de Sitter stage is final, but not the beginning that corresponds to the dependence of the energy momentum tensor (4.3) - (4.5) on the space scale:

$$a \to 0, \qquad \frac{P^2_{(h)}}{a^6} \gg \lambda,$$

$$a \to \infty, \qquad \frac{P^2_{(h)}}{a^6} \ll \lambda.$$

The observer can also discover a nontrivial motion with zero spectral energy $(\mathcal{H}_{\mathcal{P}} = 0\ )$ for an open $(k = -1)$ empty space, $\mathcal{E}_R = \mathcal{E}_d = \lambda = 0$, which is described by the equation

$$\frac{\kappa^2 P^2_{(\mu)}}{2 \cdot 6 a^6} = \frac{3}{\kappa^2 r_0^2 a^2}. \qquad (4.16)$$

For conformal time $N dt = a\, d\eta$ the solution of this equation leads to inflation:

$$a d\eta = \pm r_0 da \implies a(\eta) = a_0 \mathrm{e}^{\pm r_0 \eta}.$$

This motion is like a zero mode in a superfluid and may be given as an explanation to the hidden mass phenomenon.

5. Discussion

In the present paper, we have tried to carry out a gaugeless reduction of the phase space of gravity. The main peculiarity of this reduction is the appearance of the concepts of spectral energy and spectral time. These quantities have been obtained from the initial Einstein action with a maintenance of all surface terms, including the total time derivative. In quantum theory they correspond to nonzero phases of the wave function. Just from these phases an "Observer" forms the spectral energy and spectral time.

We have calculated these quantities for a set of simplest examples:

- In the limit of a small space scale factor a of the metric, the “Observer” is observing that the creating Universe expands from finite volume and density and is filled only with Misner anisotropic gravitons [24]. This Universe undergoes inflation with respect to spectral time. The inflation corresponds to the energy density $\propto 1/a^6$ but not to the de Sitter one.

- At the radiation stage, the reduced action has two correct limits. In the large cosmological scale limit, the spectral time coincides with the conformal one. At small scales (in the flat space-time limit) the spectral energy is nothing more but the energy of transverse photons, in contrast to the ADM scheme.

- At the stage of a dust-filled Universe, the “Observer” discovers that his spectral time, as a phase of the wave function, transforms to the classical Friedmann time, as an invariant interval (just as the observable time for a relativistic particle at rest transforms to the proper one).

- At the de Sitter stage the role of the spectral time is played by the space volume a^3

The “Observer” sees the variable character of the spectral time T_s in the process of evolution of the Universe. Who is that “Observer” whose conclusions so strongly differ from those of a modern scientist [25]?

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