

## SPINOR FIELDS IN 5-DIMENSIONAL SPACE WITH TORSION AND ELECTROWEAK INTERACTIONS

V.G.Krechet

Yaroslavl State Pedagogical University, Yaroslavl 150000, Russia

Received 25 November 1994

The dynamics of the first generation leptons in a 5-dimensional space with torsion is considered. It is demonstrated that the Lagrangian of the gravitational field of torsion and the lepton field coincides with that of the neutral sector of the electroweak theory; the space-time torsion pseudotrace corresponds to the neutral  $Z^0$  boson.

We consider the dynamics of the first generation leptons, the electrons  $e(x)$  and the left neutrinos  $\nu_L = (1 + \gamma_5)\nu/2$  in a 5-dimensional space with the torsion  $Q_{AB}^C = \Gamma_{[AB]}^C$  ( $A, B, C = 1, 2, 3, 4, 5$ ) where  $\Gamma_{[AB]}^C$  are the connection coefficients of a 5-dimensional space with torsion:

$$\Gamma_{[AB]}^C = \left\{ \begin{matrix} C \\ AB \end{matrix} \right\} + Q_{.AB}^C + Q_{.AB}^C + Q_{BA}^C. \quad (1)$$

Assuming, as usual, that at small distances of the scale of particle physics the space-time curvature effects can be neglected, one can assert that the 5-dimensional metric is the Minkowski one,  $G_{AB} = \eta_{AB} = \text{diag}(1, 1, 1, -1, 1)$ , so that the connection coefficients are determined only by the torsion:  $\Gamma_{[AB]}^C = Q_{.AB}^C + Q_{.AB}^C + Q_{BA}^C$ .

The Lagrangian of the lepton fields can be presented as a generally covariant generalization of the corresponding special-relativistic Lagrangian:

$$L_\psi = \frac{\hbar c}{2} \left[ \nabla_A \bar{e} \gamma^A e - \bar{e} \gamma^A \nabla_A e - 2\mu_0 \bar{e} e + \nabla_A \bar{\nu}_L \gamma^A \nu_L - \bar{\nu}_L \gamma^A \nabla_A \nu_L \right]. \quad (2)$$

Here  $\nabla_A \psi$  is the covariant derivative of the spinor functions,  $\psi = (e, \nu)$ :  $\nabla_A \psi = \partial_A \psi - \Gamma_A \psi$  where  $\Gamma_A$  is the spinor connection in the 5-dimensional space:

$$\Gamma_A = \frac{1}{4} \gamma^C (\Gamma_{AC}^B \gamma_B - \partial_A \gamma_C), \quad (3)$$

while the Dirac matrices  $\gamma_A = (\gamma_\alpha, \gamma_5)$ ,  $\alpha = 1, 2, 3, 4$ , satisfy the the fundamental condition connecting the space with the spin:

$$\gamma_A \gamma_B + \gamma_B \gamma_A = 2G_{AB} \quad (4)$$

The gravitational field Lagrangian is chosen as the 5-dimensional generalization of that of general relativity, i.e., the scalar curvature:  $L_g = -^5R/2\kappa$ , so that

the total Lagrangian of the system which consists of geometry and matter is a sum of the corresponding Lagrangians, i.e., the geometric Lagrangian and the spinor Lagrangian (2):

$$L = -^5R/2\kappa + \frac{\hbar c}{2} \left[ \nabla_A \bar{e} \gamma^A e - \bar{e} \gamma^A \nabla_A e - 2\mu_0 \bar{e} e + \nabla_A \bar{\nu}_L \gamma^A \nu_L - \bar{\nu}_L \gamma^A \nabla_A \nu_L \right]. \quad (5)$$

As shown earlier [1], the condition (4) for the matrices is invariant with respect to the spinor connection transformation:

$$\begin{aligned} \Gamma_A &\rightarrow \Gamma_A + K_A; \\ \nabla_A \gamma_B &\rightarrow \nabla_A \gamma_B + K_A \gamma_B - \gamma_B K_A. \end{aligned} \quad (6)$$

Here  $K_A$  is an arbitrary spin-tensor which can manifestly be developed in a spin-tensor matrix basis:  $I, \gamma_A, \gamma_{[A\gamma B]}$ :

$$K_A = C_A I + \varphi \gamma_A + B_{ACD} \gamma^{[C} \gamma^{D]} \quad (7)$$

where  $C_A, \varphi, B_{ACD} = B_{[ACD]}$  are, arbitrary vector, scalar and tensor functions, respectively.

Let us introduce a 5-dimensional space-like vector field (monad)  $\lambda_A$  and the metric tensor of the 4-dimensional space-time section  $g_{AB}$  [2]:

$$G_{AB} = \lambda_A \lambda_B + g_{AB} \quad (8)$$

which satisfy the relations

$$\begin{aligned} \lambda_A \lambda_B G^{AB} &= 1; & \lambda^A g_{AB} &= 0; \\ g_{AB} g^{AB} &= 4; & \lambda_B &= G_{5B}; \\ g_{5B} &= 0; & g_{\alpha\beta} &= G_{\alpha\beta}; & g_\mu^5 &= \lambda_\mu; \\ g_\nu^\mu &= G_\nu^\mu = \delta_\nu^\mu; & g_B^5 &= 0. \end{aligned} \quad (9)$$

All the quantities considered may be projected onto the monad direction and the 4-dimensional space-time, in

particular,

$$\begin{aligned} C_A &= \lambda_A + \tilde{C}_A \\ (C &= \lambda^A C_A; \tilde{C}_A = g_A^K C_K; \tilde{C}_5 = 0); \\ Q_{ABC} &= \{Q_{\alpha\beta}, \tilde{Q}_{\alpha\beta\lambda}\}; \\ B_{ABC} &= \{B_{\alpha\beta}, \tilde{B}_{\alpha\beta\lambda}\}. \end{aligned} \quad (10)$$

Here, as the algebraic properties of the torsion tensor and the gauge tensor  $B_{ABC}$  are the same and we are dealing with a space with torsion, one may consider the 4-dimensional antisymmetric tensors  $Q_{\alpha\beta}$ ,  $B_{\alpha\beta}$  and  $\tilde{Q}_{\alpha\beta\lambda}$  to be proportional to each other:

$$\begin{aligned} B_{\alpha\beta} &= kQ_{\alpha\beta}; \quad \tilde{B}_{\alpha\beta\lambda} = (\eta - 1)\tilde{Q}_{\alpha\beta\lambda} \\ (k, \eta &= \text{const}). \end{aligned} \quad (11)$$

In addition, we adopt the cyclic dependence of the spinor (lepton) fields on the 5th coordinate:  $\psi = \psi(x^\sigma)e^{-i\alpha x_5}$ .

With these remarks taken into account, the gravitational Lagrangian, which now depends only on the torsion components, may be presented in the form

$$-{}^5R/2\kappa = -(3/\kappa)(Q_{\alpha\beta}Q^{\alpha\beta} + 2Q_\alpha Q^\alpha) \quad (12)$$

where the axial 4-vector  $Q_\alpha$  is the pseudotraces of the 4-dimensional torsion:  $Q_\alpha = \varepsilon_{\alpha\beta\lambda\sigma}Q^{\beta\lambda\sigma}$ .

The spinor Lagrangian (2) can be decomposed into the kinetic part and the interaction Lagrangian:

$$\begin{aligned} L_\psi &= \frac{\hbar c}{2}(\partial_\alpha \bar{e}\gamma^\alpha e - \bar{e}\gamma^\alpha \partial_\alpha e) \\ &+ \frac{\hbar c}{2}(\partial_\alpha \bar{\nu}_L \gamma^\alpha \nu_L - \bar{\nu}_L \gamma^\alpha \partial_\alpha \nu_L) \\ &+ \hbar c(\mu_0 + \varphi_\theta)\bar{e}e + \hbar c(b_e - \alpha)\bar{e}\gamma_5 e \\ &+ \hbar c(b_\nu + \varphi_\nu - \alpha)\bar{\nu}_L \nu_L + L_{\text{int}}; \end{aligned} \quad (13)$$

$$\begin{aligned} L_{\text{int}} &= i\frac{\hbar c}{2}C_\alpha \bar{e}\gamma_\alpha e + \frac{3}{2}\hbar c\eta Q^\alpha \bar{e}\gamma_5 \gamma_\alpha e \\ &+ \frac{\hbar c}{2}\left(\frac{3}{2} + k\right)(Q_{\alpha\beta}(\bar{e}\gamma_5 \gamma^\alpha \gamma^\beta e + \bar{\nu}_L \gamma^\alpha \gamma^\beta \nu_L) \\ &- 3\hbar c\eta Q^\alpha \bar{\nu}_L \gamma_\alpha \nu_L). \end{aligned} \quad (14)$$

Here  $\varphi_\theta$ ,  $b_e$ ,  $\varphi_\nu$  and  $b_\nu$  are gauge coefficients for the electron connection and the neutrino spinor connection. A choice of these coefficients enables one to obtain the observed values of the electron and neutrino masses:  $\mu_0 + \varphi_\theta = \mu_e$ ;  $b_e - \alpha = 0$ . Assuming that the constant  $\alpha$  in (14) is equal to  $-3/2$ , we remove the interaction of the spinor fields through an anomalous magnetic moment (AMM):  $Q_{\alpha\beta}\psi\gamma_5\gamma^\alpha\gamma^\beta\psi$ . As a result, the lepton spinor Lagrangian in the 5-dimensional space with torsion takes the form

$$\begin{aligned} L_\psi &= \frac{\hbar c}{2}(\partial_\alpha \bar{e}\gamma^\alpha e - \bar{e}\gamma^\alpha \partial_\alpha e + \partial_\alpha \bar{\nu}_L \gamma^\alpha \nu_L - \bar{\nu}_L \gamma^\alpha \partial_\alpha \nu_L) \\ &+ \hbar c\mu_e \bar{e}e + i\hbar cC_\alpha \bar{e}\gamma^\alpha e \\ &+ \frac{3}{2}\hbar c\eta Q^\alpha \bar{e}\gamma_\alpha \gamma_5 e - 3\hbar c\eta Q^\alpha \bar{\nu}_L \gamma_\alpha \nu_L. \end{aligned} \quad (15)$$

Comparing it with the Lagrangian of the lepton fields interacting with the neutral vector fields (electromagnetic,  $A_\alpha$ , and neutral  $Z^0$  boson) in the electroweak interaction theory,

$$\begin{aligned} L_\psi &= \frac{\hbar c}{2}(\partial_\alpha \bar{e}\gamma^\alpha e - \bar{e}\gamma^\alpha \partial_\alpha e + \partial_\alpha \bar{\nu}_L \gamma^\alpha \nu_L - \bar{\nu}_L \gamma^\alpha \partial_\alpha \nu_L) \\ &+ \hbar c\mu_e \bar{e}e + ie_0 A^\alpha \bar{e}\gamma_\alpha e \\ &+ \frac{g}{4}Z^\alpha \bar{e}\gamma_\alpha \gamma_5 e - \frac{g}{2}Z^\alpha \bar{\nu}_L \gamma_\alpha \nu_L \\ &+ \frac{3g_1^2 - g_2^2}{4g}Z^\alpha \bar{e}\gamma_\alpha e \end{aligned} \quad (g = \sqrt{g_1^2 + g_2^2}), \quad (16)$$

we see that, under the condition  $3g_1^2 = g_2^2$  between the electroweak interaction constants, which leads to the value of the Weinberg angle  $\theta_W = 30^\circ$  close to the observed one ( $\sim 29^\circ$ ), the two Lagrangians coincide up to notations. This makes possible the following identification:  $\hbar cC_\alpha = e_0 A_\alpha$ ;  $\frac{3}{2}\hbar c\eta Q^\alpha = \frac{g}{4}Z^\alpha$ . One may conclude that the torsion pseudotraces of the 4-dimensional space-time  $Q^\alpha = \varepsilon^{\alpha\beta\lambda\sigma}\tilde{Q}_{\beta\lambda\sigma}/6$  is the neutral  $Z^0$  boson in the electroweak interaction; simultaneously the present geometric theory fixes a value of the Weinberg angle close to the observed one,  $\theta_W = 30^\circ$ . In addition, as we have noticed before [3], the Lagrangians (15) and (16) without the spinor field mass terms ( $\mu_e = 0$ ) are invariant with respect to the local group  $U(1) \times \tilde{U}(1)$  where  $U(1)$  is the phase rotation group and  $\tilde{U}(1)$  is the group of  $\gamma_5$ -transformations of the spinor field:

$$\begin{aligned} \delta\psi &= i\varepsilon(x)\psi + \omega(x)\gamma_5\psi; \\ \delta A_\alpha &= \frac{1}{e_0}\partial_\alpha\varepsilon; \quad \delta Q_\alpha = \frac{2}{3\hbar c\eta}\partial_\alpha\omega \\ (\delta Z_\alpha &= \frac{4}{g}\partial_\alpha\omega). \end{aligned} \quad (17)$$

One may thus conclude that the electromagnetic field  $A_\alpha$  and the space-time torsion pseudotraces field (or the  $Z^0$  boson field) are gauge fields of the local group  $U(1) \times \tilde{U}(1)$  (17) and simultaneously those of the group  $U(1) \times SU(2)$ , while the whole neutral sector of the electroweak interaction theory is the gauge theory of the  $U(1) \times \tilde{U}(1)$  group (prior to the spontaneous symmetry breakdown inducing the masses):

$$\begin{aligned} L &= -\frac{1}{4}(F_{\alpha\beta}^2 + Z_{\alpha\beta}^2) + L_\psi(\partial_\alpha\psi) \\ &+ L_{\text{int}}(\bar{e}, \bar{\nu}_L, Z_\alpha, A_\alpha). \end{aligned} \quad (18)$$

This theory admits a total geometric formulation in the 5-dimensional space with torsion:

The gravitational Lagrangian (12),  ${}^5R/(2\kappa) = (3/(2\kappa))(Q_{\alpha\beta}^2 + 2Q_\alpha^2)$ , can be expressed, taking into account the correspondence between the torsion pseudotraces  $Q_\alpha$  and the intermediate  $Z$  boson, in terms

of the vector and pseudovector potentials:

$$\frac{3}{2\kappa}(Q_{\alpha\beta}^2 + 2Q_\alpha^2) = -\frac{1}{4}M_{\alpha\beta}^2 - \frac{64}{27} \frac{\alpha}{\kappa\hbar c\eta^2} Z_\alpha^2$$

$$(\alpha = \frac{e_0^2}{\hbar c}). \quad (19)$$

Here we have introduced the gauge  $(U(1) \times \check{U}(1))$  invariant tensor  $M_{\alpha\beta}$ :

$$\sqrt{\frac{3}{2\kappa}} Q_{\alpha\beta} = \frac{1}{2} M_{\alpha\beta} = \frac{1}{2} (F_{\alpha\beta}^2 + \check{Z}_{\alpha\beta}^2)$$

$$(\check{Z}_{\alpha\beta} = \frac{1}{2} \varepsilon_{\alpha\beta\lambda\sigma} Z^{\lambda\sigma}) \quad (20)$$

which is set equal to the antisymmetric torsion tensor (the first equality sign in (20)). The Lagrangian (19) of geometric origin coincides up to a divergence term with the neutral vector field Lagrangian in the Weinberg-Salam theory:  $L = -\frac{1}{4}(F_{\alpha\beta}^2 + \check{Z}_{\alpha\beta}^2)$ . Noteworthy, it also contains the mass term for the  $Z$  boson:  $\frac{64}{27} \frac{\alpha}{\kappa\hbar c\eta^2} Z_\alpha Z^\alpha = (\frac{m_z c}{\hbar})^2 Z_\alpha Z^\alpha$  ( $\eta \sim 10^{16}$ ), while the electromagnetic field remains massless. The resulting total Lagrangian of the lepton field dynamics in the 5-dimensional space with torsion coincides with the Lagrangian for the neutral sector of the electroweak interaction theory (at  $\theta_W = 30^\circ$ ):

$$-\frac{1}{4} M_{\alpha\beta} M^{\alpha\beta} + \left(\frac{m_z c}{\hbar}\right)^2 Z_\alpha Z^\alpha$$

$$+ \frac{\hbar c}{2} (\partial_\alpha \bar{e} \gamma^\alpha e - \bar{e} \gamma^\alpha \partial_\alpha e - 2\mu_e \bar{e} e)$$

$$+ \frac{\hbar c}{2} (\partial_\alpha \bar{\nu}_L \gamma^\alpha \nu_L - \bar{\nu}_L \gamma^\alpha \partial_\alpha \nu_L)$$

$$+ ie_0 A^\alpha \bar{e} \gamma_\alpha e + \frac{g}{4} Z^\alpha \bar{e} \gamma_\alpha \gamma_5 e - \frac{g}{2} Z^\alpha \bar{\nu}_L \gamma_\alpha \nu_L. \quad (21)$$

As the Lagrangian (21) is invariant with respect to the local  $U(1) \times \check{U}(1)$  group (when the particle masses are absent), the corresponding lepton and vector field dynamics can be formulated in a dually symmetric form. A dually symmetric electrodynamics for electrons can be obtained by varying the action with the Lagrangian (21) in  $e$ ,  $Z_\alpha$  and  $A_\alpha$ , which yields a set of equations for the electron field, the  $Z^0$  boson field and the electromagnetic field, symmetric with respect to the duality transformation (provided the  $Z$  boson mass is zero)  $M_{\alpha\beta} \rightarrow \check{M}_{\alpha\beta}$ ,  $J_\alpha \rightarrow \check{J}_\alpha$ :

$$(i) \quad \partial_\alpha M^{\alpha\beta} = J^\beta;$$

$$(ii) \quad \partial_\alpha \check{M}^{\alpha\beta} = \check{J}^\beta;$$

$$(iii) \quad \gamma^\alpha (\partial_\alpha - ie_0 A_\alpha - i\frac{g}{4} \gamma_5 Z_\alpha) e = 0$$

$$(J_\alpha = e_0 \bar{e} \gamma_\alpha e; \check{J}_\alpha = \frac{g}{4} \bar{e} \gamma_\alpha \gamma_5 e) \quad (22)$$

The following conclusions can be made.

1. The geometric theory of lepton dynamics in the 5-dimensional space with torsion is equivalent to

the theory of interaction of leptons with the electromagnetic and  $Z$  boson fields, i.e., coincides with the neutral sector of the electroweak interaction theory.

2. The neutral  $Z^0$  boson can be identified with the space-time torsion pseudotraces, both of them being the gauge fields of the local group of  $\gamma_5$  transformations.
3. The neutral sector of the weak interaction theory, as well as lepton dynamics in the 5-dimensional space with torsion, can be presented as dually symmetric electrodynamics.

## References

- [1] V.G.Krechet. In: Abstracts of Int. School-Seminar "Multidimensional Gravity and Cosmology" (Yaroslavl, 1994), Moscow 1994, p.24.
- [2] Yu.S.Vladimirov. "Frames of Reference in Gravitation Theory", Moscow, Energoatomizdat 1982.
- [3] V.G.Krechet, Izvestiya Vuzov, Fizika, 1988, No.3, 114.