

# CHIRAL NONLINEAR SIGMA MODELS AND COSMOLOGICAL INFLATION

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A self-gravitating chiral nonlinear sigma model with a potential term is considered in the context of inflation scenarios as a generalisation to self-interacting scalar field theory. The complete set of new exact solutions for the two-component sigma model in the framework of spatially flat Friedmann-Robertson-Walker and de Sitter universes is obtained in the case of ultrastiff matter.

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## 1. Introduction

Various versions of the inflationary universe scenario have exploited the theory of a self-interacting scalar field (SSF) coupled to gravity within the context of cosmological spaces. The non-linear scalar field theory, possessing a self-interaction potential  $V(\phi)$ , can be regarded as the best effective theory which describes particle physics phenomena in the early universe such as spontaneous breakdown and restoration of the GUT gauge symmetry [1].

On the basis of SSF minimally coupled to Einstein's gravity in the framework of isotropic and homogeneous Friedmann-Robertson-Walker (FRW) universes the models of "old inflation" [2], "new inflation" [3] and chaotic inflationary scenario [4] have been analyzed. Anisotropic and inhomogeneous cosmologies as well as  $(N+1)$ -dimensional space-times have also been investigated (see, for example, [5-7]).

Extended inflation [8] is based on the Brans-Dicke (BD) theory of gravity, which means in fact the existence of two scalar fields, namely, the BD field and a "matter" scalar field [9]. The BD type of scalar-tensor theory of gravity, as well as  $R^2$  gravity and induced gravity models, is conformally equivalent to Einstein's gravity coupled to a SSF [10]. Inflation scenarios in the presence of non-minimal coupling to gravity have been studied as well ( see [11] and references quoted therein).

Observing the above models, we can find that all these inflationary models are closely connected with a gravitational field produced by a single scalar field. However, the progress of grand unification and supersymmetry theories shows that a gravitational field may be produced by several (effective) scalar fields in the

very early universe. This approach has been used for a multicomponent inflationary model [12] and a double (or multiple) inflation scenario [13]. Both are based on noninteracting scalar fields with different potentials and have led to better understanding of the large-scale structure of the universe [14] and some other cosmological problems [12, 13, 15] like the horizon, flatness and graceful exit ones.

The next logical step is to introduce an interaction between scalar fields. There are at least two possibilities [16]. The first one is to use a potential term depending on all scalar fields. These type of models has been considered, for example, in Refs. [7, 17-19]. The second one is to use the chiral model (or nonlinear sigma model, NSM), where an interaction is introduced by a constraint on the values of the scalar fields [20]. The nonlinear sigma model coupled to a metric tensor of Euclidean signature (++++) has been suggested in ref.[24]. A self-gravitating NSM, based on a space-time of Lorentzian signature (----), has been introduced in [23]. In the framework of Kaluza-Klein theories the nonlinear sigma model in the context of the early universe was considered in [21].

It should be pointed out here that we have different equations and different physical sense for Kaluza-Klein theories and self-gravitating NSMs. We do not use the Einstein equations for the target space, as has been usually done in Kaluza-Klein theory. Besides, the Einstein equations for the coordinate space-time contain the energy momentum tensor of chiral fields, while in the Kaluza-Klein theory one has the vacuum Einstein equations.

Let us repeat again that the SSF theory presents only the simplest example of an effective theory describing gauge and Higgs fields (of GUT) coupled to

gravity. The nonlinear sigma model can be viewed as an effective model for GUT, like the SSF model. Besides, the NSM is richer than the SSF theory and bears many features corresponding to a gauge theory [20]. Moreover, the NSM can be reduced to the SSF theory [16, 22].

In the present article the self-gravitating nonlinear sigma model is analyzed in the framework of isotropic and homogeneous universes. As the first step of investigations of an inflation scenario, the set of the Einstein and chiral fields equations is solved. The set of exact solutions for the two-component NSM in spatially flat universes is obtained in the presence and absence of the cosmological constant. For the sake of completeness all possibilities for the sign of the cosmological constant have been taken into account because of its importance in cosmology [25].

In Section 2 the basic equations are presented and a method of solving them is described.

Section 3 contains the set of exact solutions for a spatially flat FRW universe.

In Sections 4 and 5 solutions for a spatially flat universe are presented, with positive and negative cosmological constant.

Finally, in Section 6 we discuss the solutions obtained.

## 2. The model and basic equations

Let us consider the self-gravitating NSM [23] in the presence of a cosmological constant and the potential of self-interactions for chiral fields

$$S_{(1)} = \int_{\mathcal{M}} d^m x \sqrt{g} \left\{ \frac{R + 2\Lambda}{2\kappa} + \frac{\alpha}{2} h_{AB} \varphi_i^A \varphi_k^B g^{ik} + W(\phi^C) \right\}, \quad (1)$$

where  $x = (x^1, \dots, x^m)$  are local coordinates of the base space-time, a Riemannian manifold  $(\mathcal{M}, g_{ik})$ ;  $\varphi = (\varphi^1, \dots, \varphi^n)$  is a multiplet of scalar fields which lies in a target or chiral space, a Riemannian manifold  $(\mathcal{N}, h_{AB})$ ;  $\alpha$  is a coupling constant;  $g = |\det(g_{ik})|$ ;  $\varphi_k := \varphi_{,k} := \partial_k \varphi$ ;  $R$  is the scalar curvature of the base space-time and  $\kappa$  is Einstein's constant. The summation convention over repeated indices is assumed. By varying the action (1) with respect to  $\varphi^A$  we obtain the equations of motion

$$\frac{1}{\sqrt{g}} \partial_i (\sqrt{g} g^{ik} h_{AB} \varphi_k^B) - h_{CD} \Gamma_{AB}^D \varphi_i^B \varphi_k^C g^{ik} - \frac{1}{\alpha} \frac{\partial W}{\partial \phi^A} = 0. \quad (2)$$

Here  $\Gamma_{AB}^D$  are the Christoffel symbols of the chiral manifold  $(\mathcal{N}, h_{AB})$ .

Einstein's equations for the model (1) with the energy-momentum tensor

$$T_{ik} = \alpha h_{AB} \varphi_i^A \varphi_k^B - g_{ik} \left\{ \frac{\alpha}{2} \varphi_j^A \varphi_l^B h_{AB} + W(\phi) \right\} \quad (3)$$

can be reduced to the form

$$R_{ik} = \kappa \{ \alpha h_{AB} \varphi_i^A \varphi_k^B + g_{ik} W(\phi) \} + \Lambda g_{ik}. \quad (4)$$

It should be noted here that the NSM coupled to a metric tensor with the signature  $(++++)$  has been suggested in Ref.[24]. In [24] instanton and meron solutions for the set of equations (2),(4) have been obtained in the framework of conformally Weyl spaces. The self-gravitating NSM, based on space-times of Lorentzian signature, have been introduced in [23]. This type of models rests on Einstein's conjecture that the energy-momentum tensor of matter (3) creates the gravitational field. The starting point in Kaluza-Klein theories is quite different: the dynamics of matter corresponds to Einstein's vacuum equations in a space-time of more than physical (four) dimensions. And the equivalence between the equations of general relativity and Kaluza-Klein theory should be checked in each case. After this comment let us address to a concrete model.

Let us choose the space-time metric to be that of a homogeneous and isotropic FRW universe

$$dS_{\mathcal{M}}^2 = dt^2 - e^{a(t)} \left\{ \frac{(dr)^2}{1 - kr^2} + r^2 [d\theta^2 + \sin^2 \theta d\varphi^2] \right\}. \quad (5)$$

Here  $k = 1, 0, -1$  for closed, flat and open universes, respectively.

We will use the chiral metric of the NSM, associated with the SSF theory [22], in the following form:

$$dS_{\mathcal{N}}^2 = d\phi^2 + 2P(\phi) d\chi^2, \quad \varphi^1 = \phi, \quad \varphi^2 = \chi. \quad (6)$$

Here the chiral potential  $P(\phi)$  corresponds, in some sense, to the self-interaction potential  $V(\phi)$  when  $W(\phi) = 0$ . Namely,  $P(\phi)$  should be equal to minus  $V(\phi)$  if we want to get the same equation of motion for a nonlinear sigma model as for a scalar field one. Due to this correspondence it will be useful to investigate the model  $S_{(1)}$  with  $W(\phi) = 0$ . Substituting (5) and (6) into (2) and (4), we can conclude that the chiral fields  $\phi$  and  $\chi$  should depend on the cosmological time  $t$  only. Therefore, doing some simplification, we can obtain the set of equations

$$e^a \left[ \frac{1}{2} a_{tt} + \frac{3}{4} a_t^2 \right] = \Lambda e^a - 2k, \quad (7a)$$

$$\frac{3}{2} \left[ a_{tt} + \frac{1}{2} a_t^2 \right] = -\kappa \alpha [\phi_t^2 + 2P\chi_t^2] + \Lambda, \quad (7b)$$

$$\frac{3}{2} a_t \phi_t + \phi_{tt} - \frac{dP}{d\phi} \chi_t^2 = 0 \quad (7c)$$

$$\frac{3}{2} a_t \chi_t + \chi_{tt} + \frac{d \ln P}{d\phi} \phi_t \chi_t = 0 \quad (7d)$$

where  $k = +1, 0, -1$  for closed, flat and open universes, respectively.

Let us turn our attention to the method of solving the set of equations (7). First of all, the gravitational

field can be calculated from Eq.(7a). With this result one can obtain from Eq.(7b) a relation (ansatz) between chiral fields' derivatives and the chiral potential term  $P(\phi)$ . The ansatz (7b) enables us to find the energy-momentum tensor and to reduce Eq.(7.3) to an ordinary differential equation of the first order. This reduced equation can be in general formally solved but the result will depend on our assumptions about the chiral potential and the first derivatives of the chiral fields. Therefore we will present a set of possible solutions for each case. There is no need to solve Eq.(7d) because it is a differential consequence of (7a-c).

It should be noted here that Eq.(7a) can also be reduced to the first order ordinary differential equation

$$zp \frac{dp}{dz} + 2p^2 - \Lambda z^2 + 2k = 0, \quad (p = z_t, z = e^{a/2}), \quad (8)$$

which gives the standard gravitational fields corresponding to ultrastiff matter [26] or a massless scalar field.

Thus the equation (8) gives us the scalar factor  $e^{a(t)}$  for FRW and de Sitter open, flat and closed universes within the two-component NSM (6). In the case of spatially flat universes we have got the complete set of exact solutions for the model under consideration.

### 3. Exact solutions for a spatially flat FRW universe ( $k = 0$ , $\Lambda = 0$ )

Taking into account  $\Lambda = 0$ ,  $k = 0$  in (7), we immediately obtain from (7a):

$$K^2(t) = e^{a(t)} = a_0(t + t_0)^{2/3}, \quad a_0 = \text{const.}$$

Thus the space-time metric (5) takes the form of the power law expansion

$$\begin{aligned} dS_{\mathcal{M}}^2 &= (dt)^2 - a_0(t + t_0)^{2/3} dl^2, \\ dl^2 &= (dr)^2 + r^2[(d\theta)^2 + \sin^2 \theta (d\varphi)^2]. \end{aligned} \quad (9)$$

This gravitational field is, as well as the energy-momentum tensor, independent of the chiral potential term  $P(\phi)$ . The nonvanishing components of the energy-momentum tensor can be now obtained from (7b) and read

$$\varepsilon = T_4^4 = \frac{1}{3\kappa t^2} = -T_3^3 = -T_2^2 = -T_1^1 = p. \quad (10)$$

It is obvious from (10) that the matter corresponding to this solution, gets an ultrastiff state and there is a physical singularity for the solutions presented in this section as  $t \rightarrow 0$ .

#### 3.1. Solution A

Let  $\phi_t^2 = h(t)$  and  $P = P(t)$ . The dependence of  $P$  from  $t$  may be prescribed since during a phase transition there is a dependence of the potential  $V(\phi)$  on

temperature [1] and consequently on time. Thus, obtaining  $\phi_t(t)$  from Eqs.(7), we can find  $dP/d\phi$  and then, by integration,  $P$  as a function of  $\phi$ .

The solution can be written as

$$\begin{aligned} \phi &= \int t^{-1} \sqrt{\beta^2 - \mu_0^2 P^{-1}(t)} dt, \\ \chi &= \frac{\mu_0}{\sqrt{2}} \int t^{-1} P^{-1}(t) dt, \\ \mu_0^2 &= \text{const}, \quad \beta^2 = 2(3\kappa\alpha)^{-1}, \quad P < \frac{\mu_0^2}{\beta^2}. \end{aligned} \quad (11)$$

We should remember that  $P < 0$  when  $V > 0$ . Therefore the chiral metric (6) must have the Lorentzian signature (+-).

Introducing the time dependence for the potential "by hand", we can obtain some analytic solutions from (11). For example, when  $P = -\nu^2 t^2$ ,  $\nu^2 = \text{const}$ ,

$$\begin{aligned} \phi &= -f(t) + \frac{\beta}{2} \ln \left| \frac{\beta + f(t)}{\beta - f(t)} \right| + \phi_0, \\ \chi &= \frac{\mu}{2\sqrt{2}\nu^2} t^{-2} + \chi_0. \\ f(t) &= \sqrt{\beta^2 + \mu^2/(\nu^2 t^2)}. \end{aligned} \quad (11a)$$

#### 3.2. Solution B

Let us assume that  $\phi_t = \psi(\phi)/t$  and  $P = P(\phi)$ . Then we can find the solution

$$\begin{aligned} \ln t &= \int \{\beta^2 - \mu_0^2 P^{-1}(\phi)\}^{-\frac{1}{2}} d\phi, \\ \chi &= \frac{\mu_0}{\sqrt{2}} \int P(\phi)^{-1} t^{-1} dt, \quad P < \frac{\mu_0^2}{\beta^2}. \end{aligned} \quad (12)$$

In this case the chiral potential  $P$  depends on the chiral field  $\phi$ . Substituting the concrete potential  $P(\phi)$  into the first integral in (12), one can find the dependence of  $\phi$  on  $t$  and then define the chiral field  $\chi$  from the second integral in (12).

#### 3.3. Solution C

Let us assume that  $P(\phi) = P_0 = \text{const}$ , as in the case of the "slow rolling" regime [1]. This assumption means that we investigate the theory of two non-interacting scalar fields. The solution in this case is

$$\begin{aligned} \phi &= c_1 \ln t + \phi_0, \quad \chi = c_2 \ln t + \chi_0; \\ P &= P_0, \quad c_1^2 + 2P_0 c_2^2 = \beta^2. \end{aligned} \quad (13a)$$

Note that there exists an exceptional solution when  $c_1^2 = 0$ :

$$\phi = \phi_0 \neq 0, \quad P = P_0, \quad \chi = \chi_0 + \beta(|2P_0|)^{-1/2} \ln t. \quad (13b)$$

In this case the potential is nonzero and the coupling constant  $\alpha$  should be negative when  $P_0 < 0$ . The chiral metric (6) is degenerate for the solution (13b).

### 3.4. Solution D

Let  $\chi_t = 0$  or  $\chi_i \chi^i = 0$ , that is,  $\chi$  is a null field. Then we have the standard solution for a massless scalar field for any type of the chiral potential  $P(\phi)$

$$\phi = \phi_0 + \beta \ln t, \quad \chi = \chi_0 = \text{const.} \quad (13)$$

Note that if  $P = P_0$ , the solution (14) is equivalent to (13a) with  $c_2^2 = 0$ . The chiral metric (6) is also degenerate for this solution.

### 3.5. Solution E

Let  $\chi_t = 1$ . This case, corresponding to the SSF theory [22], is described by relations

$$\begin{aligned} \phi &= 2f_1(t) + \beta \ln \left| \frac{\beta - f_1(t)}{\beta + f_1(t)} \right| + \phi_0, \\ f_1(t) &= \sqrt{\beta^2 - \sqrt{2}\mu t}, \\ \chi &= t + t_0, \quad P(\phi(t)) = \frac{\mu_0}{\sqrt{2}} \frac{1}{t}. \end{aligned} \quad (14)$$

The relations (15) may be obtained from (11) by choosing  $P(t)$ .

### 3.6. Solution F

Let us consider the case of the exponential potential

$$2P(\phi) = m e^{\lambda\phi}, \quad m = \text{const}, \quad \lambda = \text{const.} \quad (15)$$

For this case we can write the solution

$$\begin{aligned} \phi &= -\beta \ln t + 2\lambda^{-1} \ln(1 + c_3 t^{\lambda\beta}), \quad c_3 = \text{const}, \\ \chi &= 2\beta m^{-1} \int t^{(3/2)\lambda\beta-1} [1 + c_3 t^{\lambda\beta}]^{-3} dt \end{aligned} \quad (16)$$

It is clear that for some relations between  $\lambda$  and  $\beta$  (for example,  $\lambda\beta = n \in \mathbf{Z}$ ) the last integral in (17) leads to an analytic function for  $\chi$ .

It should be noted that the energy-momentum tensor for all solutions presented in this section is determined by (10).

## 4. Exact solutions for a spatially flat universe ( $k = 0, \Lambda > 0$ )

Solving Eq.(7a) with  $\Lambda > 0$ , we can obtain the exponentially expanding universe (5) in the form

$$dS_{\mathcal{M}}^2 = (dt)^2 - b_0 \{\cosh(\tau)\}^{2/3} dl^2. \quad (18)$$

Here  $\tau = t\sqrt{3\Lambda}$ ,  $b_0 = \text{const}$  and  $dl^2$  is defined in (9). We have got again a gravitational field which is independent of the target space metric (6). The non-vanishing components of the energy-momentum tensor are

$$\varepsilon = T_4^4 = -\frac{\Lambda}{\kappa \cosh^2(\tau)} = -T_3^3 = -T_2^2 = -T_1^1 = p. \quad (19)$$

The relations (19) correspond to ultrastiff matter again. Moreover, we have found a nonsingular solution, which belongs to the oscillating universe of the second kind [26]. When  $t = 0$ , the energy density is nonzero and the gravitational field (18) becomes the Minkowskian space-time. Let us now describe some special cases corresponding to the solutions of Section 3.

### 4.1. Solution A

Let  $\phi_t^2 = h_1(t)$  and  $P = P(t)$ . The solution can be written as

$$\begin{aligned} \phi &= \int \sqrt{-[\mu_0^2 P^{-1} + \beta^2]} \frac{d\tau}{\cosh \tau}, \\ \chi &= \frac{\mu_0}{\sqrt{2}} \int P^{-1} \frac{d\tau}{\cosh \tau}, \quad -\frac{\mu_0^2}{\beta^2} < P < 0. \end{aligned} \quad (20)$$

It is clear that we have to introduce the time dependence for  $P(t)$  "by hand".

### 4.2. Solution B

Let  $\phi_\tau = g_1(\phi) \cosh^{-1} \tau$  and  $P = P(\phi)$ . Then (7.2) implies that  $\chi_\tau \cosh \tau = g_2(\phi)$ . The solutions can be obtained from the relations

$$\begin{aligned} \tau &= \sinh_{-1} \tan I_1(\phi), \\ I_1(\phi) &= \int \{-[\nu^2 P^{-1} + \beta^2]\}^{-1/2} d\phi, \\ \chi &= \frac{\nu}{\sqrt{2}} \int P(\phi)^{-1} \frac{d\tau}{\cosh \tau}, \\ \nu^2 &= \text{const}, \quad -\nu^2 \beta^{-2} < P < 0. \end{aligned} \quad (21)$$

The rule for obtaining the solutions is the same as in Subsec.3.2.

### 4.3. Solution C

Let  $P(\phi) = P_0 = \text{const}$ . Under this assumption we obtain the theory of two non-interacting scalar fields. The solution is

$$\begin{aligned} \phi &= c_1 \arctan(\sinh \tau) + \phi_0, \\ \chi &= c_2 \arctan(\sinh \tau) + \chi_0, \\ c_1^2 + 2P_0 c_2^2 &= -\beta^2, \quad P_0 < -\frac{c_1^2}{2c_2^2}. \end{aligned} \quad (22)$$

### 4.4. Solution D

Let  $\chi_t = 0$  ( $c_2^2 = 0$ ). Then, for any type of the potential  $P(\phi)$ , we can find the solution

$$\phi = \beta \arctan(\sin \tau) + \phi_0, \quad \chi = \chi_0 = \text{const.} \quad (23)$$

The coupling constant  $\alpha$  should be negative.

## 4.5. Solution E

Let  $\chi_\tau = 1$ . This is a special case of Solution A (20):

$$\begin{aligned}\phi &= \int \sqrt{-[\beta^2 + \sqrt{2}\mu \cosh \tau]} \frac{d\tau}{\cosh \tau}, \\ \chi &= \tau + \chi_0, P = \frac{\mu}{\sqrt{2}} \cosh^{-1} \tau.\end{aligned}\quad (24)$$

## 4.6. Solution F

Considering the case of the exponential potential (16), one can obtain the following solution:

$$\begin{aligned}\phi &= \phi_0 - \frac{4}{\lambda\kappa\alpha} \ln \cos\left(\frac{v}{4}\sqrt{2\kappa\alpha/3}\right), \\ \chi &= c_3 \int \left\{ \cos\left(v\sqrt{2\kappa\alpha/3}\right) \right\}^{2/\kappa\alpha} \frac{dv}{\lambda}, \\ v &= \frac{\lambda}{2} \arctan(e^\tau), \quad c_3 = \text{const.}\end{aligned}\quad (25)$$

The energy-momentum tensor for all solutions presented here is defined by (19).

5. Exact solutions for a spatially flat universe ( $k = 0$ ,  $\Lambda < 0$ )

When  $\Lambda < 0$ , solution of Eq.(7a) leads to the space-time metric

$$dS_{\mathcal{M}}^2 = (dt)^2 - d_0 |\cos \eta|^{2/3} dl^2 \quad (26)$$

with  $\eta = -\sqrt{-3\Lambda}t$ ,  $d_0 = \text{const}$  and  $dl^2$  defined in (9). The nonvanishing components of the energy-momentum tensor are

$$\varepsilon = T_4^4 = -\frac{\Lambda}{\kappa \cos^2 \eta} = -T_3^3 = -T_2^2 = -T_1^1 = p. \quad (27)$$

According to (26) and (27) there is some "pulsation" of the universe (26) and the energy density  $\varepsilon$ . When  $t = 0$ , we have the Minkowskian space-time with the energy density  $\varepsilon = -\Lambda\kappa^{-1} > 0$ . Then the space-time collapses into a singularity with an infinite energy density when  $t = -\frac{\pi}{\sqrt{-3\Lambda}}\left(\frac{1}{2} + n\right)$ ,  $n = -1, -2, -3, \dots$ . Thus we have obtained an oscillating universe of the first kind [26].

Now we can obtain the solutions for chiral fields in the same special cases as in sections 3 and 4.

## 5.1. Solution A

Let  $\phi_t = \phi_t(t)$  and  $P = P(t)$ . The solution can be written as

$$\begin{aligned}\phi &= \int \sqrt{\mu^2 P^{-1} + \beta^2} \frac{d\eta}{\cos \eta}, \quad P < -\mu^2 \beta^{-2}, \\ \chi &= \frac{\mu}{\sqrt{2}} \int P^{-1} \frac{d\eta}{\cos \eta}, \quad \eta = -t\sqrt{-3\Lambda}.\end{aligned}\quad (28)$$

## 5.2. Solution B

Let  $\phi_\eta = h(\phi) \cos^{-1} \eta$  and  $P = P(\phi)$ . The solution can be obtained from the formulas

$$\begin{aligned}\eta &= \arcsin \tanh I_2(\phi), \\ I_2(\phi) &= \int \sqrt{\beta^2 - \nu_0^2 P^{-1}(\phi)} d\phi, \\ \chi &= \frac{\nu_0}{\sqrt{2}} \int P(\phi)^{-1} \frac{d\eta}{\cos \eta}, \quad P < -\nu^2 \beta^{-2} < 0.\end{aligned}\quad (29)$$

## 5.3. Solution C

Let  $P(\phi) = P_0 = \text{const}$ . Then we can write the solution

$$\begin{aligned}\phi &= c_1 \ln \left| \frac{1 + \sin \eta}{1 - \sin \eta} \right| + \phi_0, \\ \chi &= c_2 \ln \left| \frac{1 + \sin \eta}{1 - \sin \eta} \right| + \chi_0, \\ P &= P_0, \quad c_1^2 + 2P_0 c_2^2 = \beta^2, \quad -\frac{c_1^2}{2c_2^2} < P_0 < 0.\end{aligned}\quad (30)$$

## 5.4. Solution D

Let  $\chi_\eta = 0$ , i.e.  $c_2^2 = 0$  in the solution (30). Then for any type of  $P(\phi)$  we can find the solution

$$\phi = \beta \ln \left| \frac{1 + \sin \eta}{1 - \sin \eta} \right| + \phi_0, \quad \chi = \chi_0 = \text{const.} \quad (31)$$

## 5.5. Solution E

Let  $\chi_\eta = 1$ . The solution is

$$\begin{aligned}\phi &= \int \sqrt{\beta^2 - 2c_3 \cos \eta} \frac{d\eta}{\cos \eta}, \quad \chi = \eta + \chi_0, \\ P(\eta) &= \frac{c_3}{\cos \eta}, \quad c_3 = \text{const.}\end{aligned}\quad (32)$$

## 5.6. Solution F

When the chiral potential takes the exponential form (16), we can find

$$\phi = \sqrt{-3\Lambda} \ln \cosh \left\{ \frac{\beta\lambda}{4} \ln \left| \frac{1 + \sin \eta}{1 - \sin \eta} \right| \right\} + \phi_0. \quad (33)$$

The other field  $\chi$  can be found from the "ansatz" (7.2).

## 6. Discussion and remarks

The concept of inflation is based on the statement that the inflation is a temporary stage of evolution of the universe, possessing the following features:

- (i) exponential (or power law) expansion of the universe;
- (ii) vacuum-like state of matter  $p = -\epsilon$  with the absence of particles;

- (iii) spontaneous breakdown of the GUT gauge symmetry, effectively described by SSF coupled to gravity;
- (iv) a nonvanishing cosmological constant  $\Lambda$ :  $\Lambda \neq 0$ .

In the present paper it is suggested to use the nonlinear sigma model with the potential  $W(\phi)$  (see (1)) as an effective theory (Section 2), instead of the self-interacting scalar field theory (see item (iii)). It is found that in the case of the two-component NSM there exists, in accordance with item (i), an exponential and power law expansion of the universe (Sections 3 and 4). Besides, an oscillating-type universe is obtained (Section 5) when the cosmological constant is negative. To get the vacuum-like state of matter we have to assume that  $W(\phi) \neq 0$ . Examples of the gravitational fields which correspond to the vacuum-like state of matter for the massive NSM have been obtained recently [27]. In this article all solutions of the two-component NSM lead to ultrastiff matter, with the equation of state  $p = \varepsilon$ . It is found that gravitational fields for the two-component NSM are independent of the chiral metric and coincide with the gravitational fields of ultrastiff matter or a massless scalar field.

It seems that the solutions to the chiral field equations, as well as the exact solutions for the gravitational field will be useful for further physical analysis of the inflationary scenario. If we identify the potential  $V(\phi)$  and  $P(\phi)$ , one can insert  $P(\phi)$  as a given function of  $\phi$  into the solutions presented in Sections 3-5. It is also possible to use an appropriate approximation, corresponding to the version of the scenario (Eqs.(7)) presented in section 2.

It is of interest to note, that Solutions E (in Sections 3-5), corresponding to the SSF theory, show the physical sense of the auxiliary chiral field  $\chi$  as a time-like coordinate.

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