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KALUZA – KLEIN QUANTUM COSMOLOGY WITH PRIMORDIAL NEGATIVE COSMOLOGICAL CONSTANT[1]

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In many models of interest, including superstring theories, a negative vacuum energy is predicted. Although this effect is usually regarded as undesirable from a cosmological point of view, we show that this can be the basis for a new approach to the cosmology of the early Universe. In the framework of quantum cosmology (in higher dimensions) when we consider a negative cosmological constant and matter that could be dust or, alternatively, coherent excitations of a scalar field, the role of cosmic time can be understood. Then we can predict the existence of a "quantum inflationary phase" for some dimensions and a simultaneous "quantum deflationary phase" for the remaining dimensions. We discuss how it may be possible to exit from this inflation-compactification era to a phase with zero cosmological constant which allows a classical description at late times.

Most standard inflationary models [1] are based on the assumption of the existence of a positive cosmological constant, which is the source of the inflation in the very early Universe. In this process all the spatial dimensions are asymptotically exponentially enlarged. If, however, we want to use as a framework of unification of all forces of nature, a higher dimensional Kaluza-Klein model whose superstring alternative is now favored, then the idea of exponentially expanding all dimensions in the early Universe ceases to be the most interesting possibility. A more attractive alternative would be to link dynamically the smallness of the extra dimensions and the big size of the visible dimensions [2]. It appears, however, that in a classical framework it is difficult to implement this idea [3]. Therefore it is natural to study the possibility of these effects in the context of quantum cosmology [4].

We have studied [4] $1+D$-dimensional, toroidally compact Kaluza-Klein cosmology where the geometry is defined by

$$ds^2 = -dt^2 + \sum_{j=1}^{D} a_i^2(t)(dx^i)^2, \quad 0 \le x^i \le 1. \qquad (1)$$

A negative cosmological constant Λ and dust (that is, a pressureless perfect fluid state) are the sources of gravity which is described by the Einstein action. The difference between the cases of negative and positive cosmological constants is that while a positive cosmological constant generates an upside-down potential for the volume $V \equiv a_1 a_2 \cdots a_D$ of the Universe, which therefore leads to exponentially increasing volume solutions, a negative cosmological constant, in contrast, generates a potential which does not allow the volume of the Universe to expand to very large values.

In order to demonstrate the properties of a model with $\Lambda < 0$ it is convenient to choose units where $16\pi G = 1$ and make use of the new variables

$$\rho^2 = 4(D-1)V/D, \quad \theta_i = \ln(a_i/V^{1/D}), \quad i = 1, \ldots, D \qquad (2)$$

Notice that $\theta_1 + \cdots + \theta_D \equiv 0$. The independent variables that diagonalize the Lagrangian and Hamiltonian are ρ and

$$z^j = \frac{1}{\sqrt{D}+1}\sqrt{\frac{D}{2(D-1)}}[\theta_1 + \cdots + (\sqrt{D}+2)\theta_j + \cdots + \theta_{D-1}], \quad j = 1, \ldots, D-1. \qquad (3)$$

The 00-component of Einstein's equations is the constraint equation which tells us that the Hamiltonian H

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of the system is equal to 0 (below μ denotes the dust density times V):

$$H = \dot{\rho}^2 - \rho^2 \sum_{j=1}^{D-1} (\dot{z}^j)^2 + \omega^2\rho^2 - \mu = 0,$$
$$\omega^2 = -\frac{D\Lambda}{2(D+1)} > 0 \tag{4}$$

Here the dot denotes a derivative with respect to the cosmic time t. Due to the symmetry $z^i \longrightarrow z^i + c^i$ (c^i = const), the quantities $F_i = -2\rho^2\dot{z}^i$ ($i = 1,\ldots,D-1$) are conserved. Inserting $\dot{z}^i = -F_i/(2\rho^2)$ in Eq.(4), we see that in the classical case the effective potential forces the volume to collapse to zero. Such singularities may be avoided in quantum cosmology [5] and in our model, for a large class of operator orderings, the amplitude for zero volume is exactly zero [4]. Together with our choice of $\Lambda < 0$, this means that $< V >$ must be nonzero and finite.

In quantum mechanics Eq.(4) becomes $H\Psi = 0$, the so-called "Wheeler-DeWitt" (WDW) equation. The quantized degrees of freedom (ρ, z^j) constitute a D-dimensional minisuperspace. The exact solutions of the WDW equation have been explicitly found in [4]. It was found there that physically satisfactory solutions are possible provided the dust content μ is big enough. Then the possible values of $F^2 \equiv F_1^2 + F_2^2 + \cdots + F_{D-1}^2$ are quantized: $F^2 = (F^2)_n$ (here n is a nonnegative integer; for an explicit expression of $(F^2)_n$ see [4]).

One important subject is the question of operator ordering. This ambiguity affects the nature of the solutions, since it can be seen that, depending on the operator ordering we take, we obtain different additional contribution to the effective potential in the WDW equation. This contribution can have the form of an attractive or repulsive potential at $\rho = 0$. We choose to work with the operator orderings that give rise to a repulsive potential. In this case, we obtain avoidance of the cosmological singularity, in the sense that the wave function then vanishes at $\rho = 0$.

More information concerning the evolution of the Universe in this model can be obtained from the Heisenberg equations of motion, for example:

$$\frac{dz^j}{dt} = i[H\,,\, z^j] = \frac{i}{2\rho^2}\frac{\partial}{\partial z^j} \tag{5}$$

Since the z^j-dependence of Ψ can be taken to be of the form $\exp(i\sum_{j=1}^{D-1} F_j z^j)$ (due to the symmetry $z^i \longrightarrow z^i + c^i$), we get that $-i\frac{\partial}{\partial z^j} \to F_j$ and taking expectation values of both sides of Eq.(5), we get

$$\frac{d < z^j >}{dt} = - < \frac{1}{2\rho^2} > F_j \tag{6}$$

which is in agreement with the classical result. Since one can check that

$$\frac{d}{dt} < \frac{1}{2\rho^2} >= 0, \tag{7}$$

we obtain

$$<z^j> = -<\frac{1}{2\rho^2}>F_j t + \text{const}, \quad j = 1,\ldots,D-1. \tag{8}$$

Therefore

$$<\theta_i> = \alpha_i t + \gamma_i, \quad i = 1,\ldots,D$$
$$\text{where } \alpha_i, \gamma_i \text{ are constants and } \sum_{j=1}^{D} \alpha_i = 0. \tag{9}$$

In the interesting particular case when $\alpha_1 = \alpha_2 = \alpha_1 \equiv \alpha$ (isotropic evolution of three dimensions) and $\alpha_4 = \ldots = \alpha_D \equiv \tilde{\alpha}$ (isotropic extra dimensional evolution), we get [4]:

$$\tilde{\alpha} = -\frac{3\alpha}{D-3},$$
$$\alpha = \pm\frac{2\omega^2}{D[\mu - 2(n+\frac{1}{2})|\omega|]\sqrt{\frac{(D-3)(D-1)}{3}}}|F|_n\,,$$
$$|F|_n \equiv \sqrt{(F^2)_n} \tag{10}$$

Notice that, in spite of the fact that the Hamiltonian acting on the wave function vanishes, there is a cosmic time dependence of the expectation values of the variables θ^i. This seems to be strange because formally $< dQ/dt >= i < [H, Q] >= 0$ (since $H| >= 0$ and $< |H = 0$) for any Q not explicitly dependent on the time observable. This is a way of formulating the well-known problem of nonappearance of time in quantum cosmology [6]. The appearance of a cosmic time dependence in Eq.(9) does not, however, seem so strange if one notices that the wave function dependence on z^j and also the Heisenberg equations (5) are very similar to those of a free nonrelativistic particle for which a linear time evolution in the average position is also obtained. The formal argument that gave us $< dQ/dt >= 0$ fails [4] for $Q = z^j$ because H fails to be Hermitian for badly behaved states as $|z^j|$ approches infinity.

We see that for this model it is possible to describe cosmological evolution (proceeding in cosmic time) in terms of averages of quantum cosmology variables. The quantities $(\frac{D}{4(D-1)})^{1/D}\alpha_j$ are equal to the averages of the Hubble parameters which in quantum theory we define as $H_j \equiv d(\ln a_j)/dt$. Since $\sum_{j=1}^{D} < H_j >= 0$, we obtain that some dimensions have associated positive constant Hubble parameters, that is, they suffer a Quantum Inflationary process, while the remaining dimensions must have associated negative constant Hubble parameters , i.e., they suffer a Quantum Deflationary process. The average of the total volume $< V >$ remains constant. We call this phase of the Universe [4] the Quantum Inflation-Compactification (QIC) era.

The above results are also valid if, instead of dust, we introduce a massive scalar field with its homogeneous degree of freedom described quantum-mechanically [7]. This is a considerable improvement of the model, since a perfect fluid description (where dust is a special case) is only a phenomenological approach, while the description of matter as a scalar field follows from first principles.

The appearance of a negative vacuum energy has been recognized as a widespread property of many superstring models [8]. Here we have seen that a negative vacuum energy in the early Universe, which is usually regarded as a disaster [8] from a cosmological point of view, is in fact a blessing, since a phase with negative vacuum energy can be the origin of a QIC era, where the asymmetry between extra and ordinary dimensions was generated.

It is important to understand, at least at a qualitative level, how this QIC era can evolve into a phase where the cosmological constant is approximately zero and the dynamics is well approximated by the classical theory. One scenario requires only the existence of a dilaton field with a potential with two local minima: one is an absolute minimum with negative vacuum energy density, while the other has zero energy density. The QIC era is realized when we consider oscillations of the scalar field around the absolute minimum. This homogeneous degree of freedom behaves like dust, as mentioned above. The explicit solutions [7] show that the total scalar field energy (vacuum plus coherent excitations) can be positive, so that a homogeneous tunneling to the zero cosmological constant state is possible, since the volume of the Universe is finite. Furthermore, at late times the phase with approximately zero cosmological constant and growing volume can be stable against decay to the negative cosmological constant state if a suitable condition on the scalar field potential is satisfied [9].

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