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ON THERMODYNAMICS IN MULTIDIMENSIONAL COSMOLOGY[1]

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Thermodynamic formulas are found for a multidimensional cosmological model consisting of $n\,(n>1)$ factor spaces with arbitrary equations of state.

In multidimensional cosmological models (MCM) matter is often described phenomenologically as a perfect fluid with a linear equation of state. Thus it is clear that investigation of thermodynamics in these models is of special interest as it makes possible estimating parameters of the model, e.g., temperature and entropy, which can be confronted with observable ones. This problem was discussed in many papers (an extensive list of references is presented in [1–3]). A method for investigating the thermodynamics of MCM was developed in [2] for a two-component model with particular equations of state. In the present paper we extend this method to an n-component model with arbitrary equations of state in each factor space.

The topology of our MCM is

$$M = R \times M_1 \times \ldots \times M_n, \tag{1}$$

where the manifold M_i is a d_i-dimensional compact space (with an arbitrary sign of curvature) with the scale factor a_i and the volume V_i. The equation of state in each factor space M_i is

$$p_i = \xi_i \rho, \qquad i = 1, \ldots, n, \tag{2}$$

where we have $\xi_i = 0$ for dust-like matter in M_i, $\xi_i = 1/d_i$ for radiation, $\xi_1 = \ldots = \xi_n = 1/\sum_{j=1}^n d_j$ for radiation in the whole space, $\xi_i = 1$ for ultrastiff matter and $\xi_i = -1$ for vacuum.

The first law of thermodynamics can be written as follows:

$$\begin{aligned} TdS &= d\left(\rho \prod_{i=1}^n V_i\right) + \sum_{i=1}^n (p_i \prod_{j=1}^{n}{}' V_j dV_i) \\ &= \prod_{i=1}^n V_i d\rho + \rho \sum_{i=1}^n \left(\alpha_i \prod_{j=1}^{n}{}' V_j dV_i\right), \end{aligned} \tag{3}$$

where $\alpha_i = 1 + \xi_i$ and the prime denotes that the term with $j = i$ does not contribute to the product. The energy-momentum conservation, $T^{0N}{}_{;N} = 0$, leads to the constancy of the entropy [1–3]:

$$\frac{dS}{dt} = 0. \tag{4}$$

For $S =$ const we can get from (3)

$$\left.\frac{\partial \rho}{\partial V_i}\right|_{S, V_j} = -\frac{\alpha_i \rho}{V_i}, \qquad i = 1, \ldots, n; \qquad j \neq i \tag{5}$$

That is,

$$\rho = K(S) \prod_{i=1}^n V_i^{-\alpha_i}, \tag{6}$$

where $K(S)$ is an unknown function of the entropy S. Inverting (6), we can get

$$S = S(x) \tag{7}$$

with $x = \rho \prod_{i=1}^n V_i^{\alpha_i}$. Using the definition of x and Eq.(3), we obtain

$$\frac{dS}{dx} = T^{-1} \prod_{i=1}^n V_i^{-\xi_i}. \tag{8}$$

By conservation of the entropy (4), we have

$$T = B \prod_{i=1}^n V_i^{-\xi_i}, \tag{9}$$

where B is an unknown constant. Now, using Eqs.(6) and (9), we can write

$$\rho = \sigma_i \left(T \prod_{j=1}^{n}{}' V_j^{-1}\right)^{\frac{\alpha_i}{\alpha_i - 1}} \prod_{j=1}^{n}{}' V_j^{\frac{\alpha_j}{\alpha_i - 1}} \Bigg|_{\alpha_i \neq 0,1}, \tag{10}$$

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with $\sigma_i = KB^{-\alpha_i/\xi_i}$. If $k(m)$ is the number of factor spaces with $\xi_i = -1$ ($\xi_i = 0$), respectively, then Eq.(10) with (3) give $n-k-m$ differential equations

$$dS \;=\; d\left[\alpha_i\sigma_i\left(T\prod_{j=1}^{n}{}' V_j^{-1}\right)^{1/(\alpha_i-1)} \times \prod_{j=1}^{n}{}' V_j^{\alpha_j/(\alpha_i-1)} V_i\right]_{\alpha_i\neq 0,1}, \tag{11}$$

which result in

$$S \;=\; (\xi_i+1)\sigma_i\left(T\prod_{j=1}^{n}{}' V_j^{-1}\right)^{1/\xi_i} \times \prod_{j=1}^{n}{}' V_j^{(\xi_j+1)/\xi_i} V_i\Big|_{\xi_i\neq -1,0} +\; C_i, \tag{12}$$

where the integration constants C_i satisfy

$$C_i - C_j = \frac{K}{B}\left(\xi_j - \xi_i\right). \tag{13}$$

From Eq.(10) it follows that an analog of the Stefan-Boltzmann law holds only in the case $\xi_1 = \ldots = \xi_n \equiv \xi$. Then

$$\rho = \sigma T^{\frac{\xi+1}{\xi}}, \tag{14}$$

and for the temperature and the entropy we have

$$T = BV^{-\xi}, \tag{15}$$

$$S = (\xi+1)\sigma V T^{1/\xi}, \tag{16}$$

respectively, where $V = \prod_{i=1}^{n} V_i$ is the total volume of the multidimensional universe.

Perfect fluid as a phenomenological matter covers a wide range of important cases [4]. We recall that the cosmological constant, the curvatures of M_i, and a homogeneous minimally coupled massless free scalar field belong to this class [4,5]. Thus it might be worthwhile to generalize the above results to the case of a multicomponent perfect fluid with the equations of state [5]

$$p_i^{(a)} = \xi_i^{(a)}\rho^{(a)}, \qquad i = 1,\ldots,n;\; a = 1,\ldots,m \tag{17}$$

subject to the conservation law constraint [5]

$$T_{N;M}^{M(a)} = 0, \qquad a = 1,\ldots,m. \tag{18}$$

Eqs.(18) imply that the first law of thermodynamics takes place for each component:

$$dQ^{(a)} \;\equiv\; T^{(a)}dS^{(a)} = \prod_{i=1}^{n} V_i d\rho^{(a)} + \rho^{(a)}\sum_{i=1}^{n}\left(\alpha_i^{(a)}\prod_{j=1}^{n}{}' V_j dV_i\right), \quad a = 1,\ldots,m. \tag{19}$$

The conservation law constraint (18) leads to the constancy of the entropy

$$\frac{dS^{(a)}}{dt} = 0, \qquad a = 1,\ldots,m. \tag{20}$$

It is easily seen in this case that Eqs.(5)–(16) are valid for each component separately. For example,

$$\rho^{(a)} = K^{(a)}\prod_{i=1}^{n} V_i^{-\alpha_i^{(a)}}, \tag{21}$$

$$T^{(a)} = B^{(a)}\prod_{i=1}^{n} V_i^{-\xi_i^{(a)}} \tag{22}$$

etc. Eq.(21) was also obtained in [4,5] from the continuity equation. Of course, for some components (e.g., for the perfect fluids associated with the curvatures of M_i) the temperature and entropy should be understood in a generalized sense as effective quantities.

The formulas obtained here can be useful for studies of multidimensional cosmological models with a perfect fluid as a matter source [6].

References

[1] Y. Tosa, Phys. Rev. D33 (1986), 2326.

[2] Y. Tosa, Phys. Lett. B174 (1986), 156.

[3] M. Szydlowski and B. Lapeta, Gen. Relat. & Grav. 21 (1989), 383.

[4] U. Bleyer and D.-E. Liebscher, Ann. d. Physik (Lpz.) 44 (1987), 81; U. Bleyer, D.-E. Liebscher, H.-J. Schmidt and A. Zhuk, Wissenschaftlische Zeitschrift (Jena) 39 (1990), 22.

[5] V.D. Ivashchuk and V.N. Melnikov, Int. J. Mod. Phys. D3 (1994), No.4; V.D. Ivashchuk and V.N. Melnikov, Billiard Representation for Multidimensional Cosmology with Multicomponent Perfect Fluid near the Singularity, Class. Quant. Grav. (1995), in press.

[6] U. Bleyer and A. Zhuk, Kasner-like, Inflationary and Steady-state Solutions in Multidimensional Cosmology, Int. J. Mod. Phys. (1995), in press.