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DETERMINATION OF STRONG INTERACTION CONSTANTS ON THE BASIS OF MESON MASS SPECTRA

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A method for determination of strong interaction constants is considered. The method is based on the relativistic potential model with phenomenological selection rules. The strong coupling constant evaluation error in a nonperturbative region is $5 \cdot 10^{-2}$. Within this error the flavour independence hypothesis for the confinement potential is confirmed for all known quark flavours. Possible techniques for further modification of this method to improve its accuracy are discussed.

In the framework of the quantum theories of fundamental interactions some quantities are known which characterize the properties of matter. These quantities are called quantum fundamental physical constants. According to Ref.[1], we may divide all the fundamental physical constants into four groups and from those groups we may select the constants of interaction like α, α_{S}, Λ_{QCD} and the constants of elementary constituents of matter like $m_{\mathrm{e}}, m_{\mathrm{quark}}$. At present $\alpha_{\mathrm{S}}, \Lambda_{\mathrm{QCD}}$ and m_{quark} are known with the worst accuracies, especially α_{S} in the nonperturbative region at large distances between the strongly interacting particles.

In the present paper we describe a method for evaluating the strong interaction constants with high accuracy on the basis of the existing data for meson mass spectra. This method is based on the relativistic model for quasi-independent quarks, which is in turn a ground for the so-called bag models for hadrons [2].

The relativistic quasi-independent quark model is suitable for hadrons with any number of both fermion and boson constituents. The main statement of this model is the assumption that some properties of hadrons can be reproduced with the help of a system of independent constituents (or quasi-independent ones with weak residual interactions), which move in an external field.

In the framework of our approach [3] based on the independent quark model, a hadron is a composite system consisting of a white gluon field which confines the quarks together and any number of light or heavy quarks with a residual colour interaction. In the rest frame of a hadron we work in the equal time approximation, so that the time coordinates of the constituents are all equal to each other: $t_0 = t_1 = \ldots = t_n$, where t_0 is the time coordinate of the confining gluon field and $t_1, \ldots, t_n$ are the time coordinates of the quarks or antiquarks. Then the external potential, which has a Lorentz invariant form, becomes static.

We consider an extended space-time symmetry group G, which includes the Poincare group as a subgroup, and for this reason we call the quark fields generalized ones. Among different G we select the minimal one, $\mathrm{G}_{\mathrm{min}}$, which is a special pseudounitary rank 3 group.

The motion of each constituent is described by a one-particle equation of Dirac type with a static QCD-motivated potential. Further on we consider only $(\bar{q}q')$-mesons, for which we may evaluate quite correctly the whole mass spectrum for both radial and orbital excitations. For $(\bar{q}q')$-mesons we may include a residual quasi-Coulomb interaction in an external potential, for which we choose a QCD-motivated spherically symmetric form:

$$U_{QCD} = -4\alpha_{\mathrm{S}}/3r + \sigma r \tag{1}$$

where α_{S} is the perturbative strong interaction coupling constant and σ is the nonperturbative one, which is sometimes called string tension.

For $(n^{2S+1}L_J)$-states of $(\bar{q}q')$-mesons the values of angular momentum for each constituent particle j_1 and j_2 must obey the following rules:

$$\begin{array}{ll} \text{if } J = L + S, & \text{then } j_1 = j_2 = J + 1/2, \\ \text{if } J \neq L + S,\ \mu_1 \leq \mu_2, & \text{then } j_1 = j_2 + 1 = J + 3/2, \end{array}$$

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(2)

where $\mu_i,\ i=1,2$ are the current quark masses, while the radial quantum numbers are equal: $n_1^r = n_2^r = n-1$.

Let us define the mass spectral function $E(n^r,j)$ of the quark/antiquark or quark/antiquark term inside a meson in the following form:

$$E(n^r,j) = c + (\lambda + \mu^2 - k\sigma)^{1/2}, \tag{3}$$

where c and k are phenomenological constants, μ is a current quark mass, λ is the eigenvalue of the model radial equation which, with the potential (1), has the form:

$$\Phi(r)'' + \lambda\Phi(r) = \Big[L'(L'+1)/r^2 - 4\alpha_S(\lambda+\mu^2-k\sigma)^{1/2}/3r + \mu\sigma r + \sigma^2 r^2/4\Big]\Phi(r) \tag{4}$$

where $L' \sim L$ and may include some phenomenological terms. The meson mass $M(n^r, J^{PC})$ is equal to $E_1(n_1^r, j_1) + E_2(n_2^r, j_2)$.

In the case of light mesons we may neglect with a good accuracy the small values of a quark/antiquark mass and a residual quasi-Coulomb quark-antiquark interaction. Then Eq. (4) can be solved exactly and we have an analytic formula for the meson mass $M(n^r, J^{PC})$, which is a generalization of the well-known Chew-Frautschi and Veneziano-Nambu formulae. If we restrict ourselves to 1^{--}-mesons, in order to use the numerous high precision data for the above heavy mesons [4], then for light mesons we obtain the approximate analytic formula

$$E(n^r,j) = c + \kappa(2n^r + L + j - k')^{1/2}, \tag{5}$$

where the phenomenological parameters are $c \approx 0, k' \approx 1/2, \kappa \approx 0.38GeV$.

For the heavy mesons we must find $\lambda(n^r,j)$ numerically, for example, using the Numerov three-point method for solving Eq. (4). However, for the singular potential $\sim r^{-\nu}$ the main loss of accuracy occurs in the neighbourhood of $r=0$. In this case to improve the evaluation accuracy of the quark term $E(n^r,j)$ we may use the polynomial extrapolation of the wave function near $r=0$.

We have, however, the main trouble with the phenomenological terms c and k'. To exclude the influence of the uncontrolled phenomenological terms c and k' in the model considered, the following relations may be used:

$$\frac{\lambda_l - \lambda_n}{M_l - M_n} = \frac{\lambda_i - \lambda_j}{M_i - M_j} + \frac{1}{4}(M_l + M_n - M_i - M_j)$$

where M_k is the value of the $(\bar{q}q)$-meson mass with given J^{PC}, λ_k is the eigenvalue for Eq.(4), which corresponds to M_k.

By a tentative calculation, the value of σ, where σ is the string tension, is equal to $0.20 \pm 0.01 GeV^2$ and this value is the same for all known quark flavours. So within the error ≤ 0.05 the flavour independence hypothesis for the confinement potential is confirmed [5]. To our knowledge, the accuracy of this evaluation of σ is the best as compared with other models.

It is known that both the phase transition from hadron matter to quark-gluon plasma and the plasma reactions depend essentially on the value of the confinement force which acts on the plasma boundary. If we extend the flavour independence hypothesis, previously formulated for confinement potential inside hadrons, to include quark-gluon plasmas, we must conclude that the value of σ should be universal for both hadrons and quark-gluon plasma volumes. Thus we may use the value $\sigma = 0.20 \pm 0.01\,\mathrm{GeV}^2$ for the evaluation of the phase transition temperature and the plasma surface radiation intensity [6].

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