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PHYSICAL FOUNDATIONS OF LINEAR ALGEBRA AND EUCLIDEAN GEOMETRY

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A unified programme is suggested for axiomatic building of linear algebra and all possible global geometries: Euclidean geometry, the Lobachevsky and Riemann constant curvature geometries, even- and odd-dimensional symplectic geometries and so far unknown new geometries, emerging in a natural way from the general theory of physical structures. The latter studies a sufficiently broad class of possible relations, first and foremost those existing between physical objects and admitting a simple physical interpretation.

1. Introduction

The present work is a concrete realization of the ideas repeatedly put forward by Yu.S. Vladimirov, who claimed that "the classical views of space-time, along with all geometric notions, are not primary. The geometric approach reflects an important, historically conditioned, but temporary stage in the development of our ideas of the physical picture of the world. It is to be replaced by a new stage, with a search for more primary views, so that the classical space-time notions should be their consequences" [1].

Thus, for instance, to perceive and understand the characteristic features of the structures underlying relativity, it is necessary to compare them with similar structures forming the basis of linear algebra and Euclidean geometry. To this end, however, one must above all understand, what is the main, depth content of these well-known branches of mathematics. This can be done by looking at the foundations of geometry and linear algebra from the standpoint of fundamentally new, very general ideas.

The well-known mathematician V.F. Kagan wrote the following about the importance of studying the foundations of geometry: "It is difficult to name a single outstanding geometrist who would not pay due attention to the foundations of geometry. Descartes, Leibniz, Lagrange, Fourier, Gauss, Cauchy, Ampère — all those creators of new mathematics, the people with so diverse scientific, philosophical and political views, all of them, reflected on the foundations of geometry, trying, by N.I. Lobachevsky's expression, to shed light on those "dark notions, used now, following Euclid, to begin geometry"... I would like just to note here that the volume and content of geometry, and maybe mathematics as a whole, have deeply changed due to the ideas brought in by the reflections on its foundations and that those ideas have exerted a great influence on the whole architecture of theoretical natural science. One may add that the mission of science is, in particular, to unify many facts to a single system, to indicate an inherent connection between them, to establish the so-called principles of science, i.e., the facts that cause all the rest; to reveal the real content of its assumptions, without diminishing this content by rough intuition and not ascribing them by tradition what had not ever been there; to account for each term used instead of calling clear all that is habitually repeated. The mission of science is, after all, to reveal the source where its truths are drawn: not certainly those which are logically derived from others and hence originate from the latter, but those which are premises for all the rest, the so-called bases of science" [2].

The task of revision and improvement of the notion apparatus of modern physics turned out to be very difficult, both in the aspect of its "mathematical instruments" and apparently in its methodological comprehension. The point is that a revolution in physics turns out to be fruitful only if allows one to reveal concrete mathematical tools able to formulate the basic, deepest laws of the reality [3].

2. Foundation of physical structures theory

As known, a set is called a space if there is a special *type of relations* between its elements, or, in other words, a set where a certain *structure* is specified.

There is a special theory, the *physical structures theory*, studying a sufficiently broad class of possible relations which take place above all between physical objects [4, 19].

On the basis of the physical structures theory, one can build a new, more natural and therefore the most adequate axiomatics underlying linear algebra and all possible global geometries [13, 18, 25].

Although there are more than a hundred of papers published on the physical structures theory, three Ph.D theses and one doctoral (Dr.Sci.) dissertation have been defended (G.G. Mikhailichenko [11]), this theory remains little-known, since from the very beginning it was aimed at solving unusual, to a certain extent even "heretical" problems emerging in the field of foundations of physics and lying outside the frames of the traditional problems considered by academic science.

Therefore it seems to me to be appropriate to present the statement of the problem and the main results of the physical structures theory.

2.1. Initial notions

The initial notions of the special physical structures theory on a set of physical objects of the same nature are:

The set of physical objects of the same nature, being a manifold of a certain dimensionality, unspecified at the outset,

$$\mathcal{A} = \{a, b, \ldots\}$$

and a *representer*, a mapping putting into correspondence a real number to each ordered pair of physical objects $(a,\ b)$:

$$w : \mathcal{A} \times \mathcal{A} \longrightarrow R$$

$$(a, b) \longmapsto w(a, b).$$

A representer is a physical quantity measured by experiment and admitting a simple physical interpretation.

Let us specify a positive integer $r \geq 2$, the *rank of a physical structure* and form from physical objects, belonging to the set A, two arbitrary finite subsets, *two corts* [1], each consisting of r physical objects:

$$M = \{a_1, \ldots, a_r\}, \qquad N = \{b_1, \ldots, b_r\}$$

The physical objects belonging to the corts M and N are of the same nature and are in certain relations with each other. These relations are characterized by r^2 representers:

$$\begin{matrix} w(a_1, b_1) & \cdots & w(a_1, b_r) \\ \cdots & \cdots & \cdots \\ w(a_r, b_1) & \cdots & w(a_r, b_r) \end{matrix} \qquad (1)$$

The set of all r^2 representers forms a matrix $\|w_{M,N}\|$ which characterizes the relations between the two corts M and N, treated as new physical objects of the same nature.

[1] A cort is an abbreviation of the word "cortège"

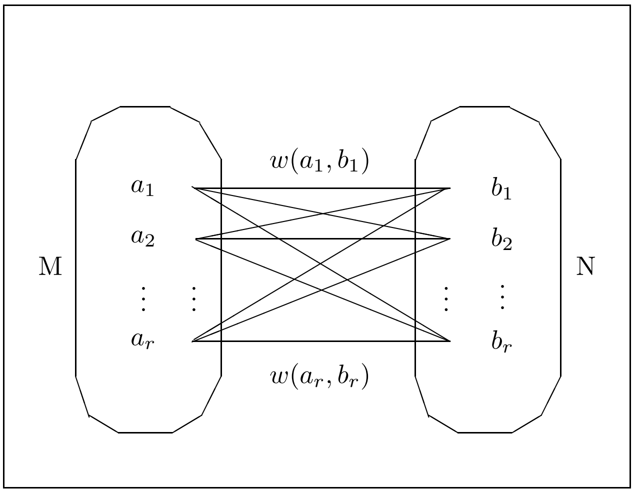

Figure 1: r^2 representers characterizing the relations between physical objects united in two arbitrary corts M and N.

2.2. Definition of a physical structure

The main and, in essence, the only requirement underlying the theory of physical structures of rank r on a single set $\mathcal{A}$ is the following:

We will say that a physical structure of rank r is specified on a set $\mathcal{A}$ of physical objects if, for an arbitrary choice of corts $M \subset \mathcal{A}$ and $N \subset \mathcal{A}$, a characteristic point with r^2 coordinates (1) in the corresponding r^2-dimensional arithmetic space R^{r^2} lies on a certain r^2-1-dimensional hypersurface, or, in other words, if there is such a smooth function of r^2 variables

$$\begin{matrix} \Phi(u_{11}, \ldots, u_{1r}, \\ \ldots\ldots\ldots\ldots \\ u_{r1}, \ldots, u_{rr}) \end{matrix},$$

that for any

$$M = \{a_1, \ldots, a_r\} \quad \text{and} \quad N = \{b_1, \ldots, b_r\}$$

the following relation is valid:

$$\begin{matrix} \Phi(w(a_1, b_1), \ldots, w(a_1, b_r), \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ w(a_r, b_1), \ldots, w(a_r, b_r)) \end{matrix} = 0 \qquad (2)$$

3. Main results

Here I present the results obtained by myself and my students, Dr. Sci., Prof. Gennady G. Mikhailichenko and Senior Researcher Vladimir Kh. Lev (Novosibirsk), and published in a large series of papers dedicated to the creation and development of physical structures theory. To get a certain idea of that theory, the unconventional nature of its tasks and the unusual mathematics that is central to its formalism, it is desirable to get acquainted with some of these papers [4–26].

We showed that there are, up to an evident equivalence, *two and only two* pairs of functions

$$w(a,b) \quad \text{and} \quad \Phi(u_{11},\dots,u_{rr})$$

satisfying the basic relation that determines a physical structure of rank r on a single set A:

A. The *vector type* solution, creating a "multiplicative" metric [4, 7, 12].

This solution has the form of an asymmetric scalar product

$$w_1(a,b) = x^1(a)\xi_1(b) + \ldots + x^{r-1}(a)\xi_{r-1}(b)$$

and the function

$$\Phi_1(u_{11},\dots,u_{1r}, \;\ldots\ldots\;, u_{r1},\dots,u_{rr}) = \Gamma_{1\ldots r;1\ldots r}(u) = \begin{vmatrix} u_{11},\dots,u_{1r} \\ \ldots\ldots\ldots \\ u_{r1},\dots,u_{rr} \end{vmatrix}$$

has the form of a Gram determinant.

It is easily directly verified that $w_1(a,b)$ and $\Gamma_{1\ldots r;1\ldots r}(u)$ indeed satisfy the above relation (2), representing the determinant $\Gamma_{a_1\ldots a_r;b_1\ldots b_r}(w_1)$ in the form of a product of two determinants, each being identically equal to zero [23]:

$$\Gamma_{a_1\ldots a_r;b_1\ldots b_r}(w_1) = \begin{vmatrix} w_1(a_1,b_1)\ldots w_1(a_1,b_r) \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ w_1(a_r,b_1)\ldots w_1(a_r,b_r) \end{vmatrix}$$

$$= \begin{vmatrix} x^1(a_1)\ldots x^{r-1}(a_1)\;0 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ x^1(a_r)\ldots x^{r-1}(a_r)\;0 \end{vmatrix} \cdot \begin{vmatrix} \xi_1(b_1)\;\ldots\;\xi_1(b_r) \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \xi_{r-1}(b_1)\ldots\xi_{r-1}(b_r) \\ 0\;\;\ldots\;\;0 \end{vmatrix} \equiv 0.$$

B. The *point-type* solution, creating an *additive* metric [4, 7, 12].

This solution has the form of an asymmetric scalar product "with a broken-up tail"

$$w_1(a,b) = x^1(a)\xi_1(b) + \ldots + x^{r-2}(a)\xi_{r-2}(b) + x^{r-1}(a) + \xi_{r-1}(b)$$

and a function

$$\Phi_2(u_{11},\dots,u_{1r}, \;\ldots\ldots\;, u_{r1},\dots,u_{rr}) = \Gamma^1_{1\ldots r;1\ldots r}(u) = \begin{vmatrix} 0 & 1 & \ldots & 1 \\ -1 & u_{11} & \ldots & u_{1r} \\ \ldots & \ldots & \ldots & \ldots \\ -1 & u_{r1} & \ldots & u_{rr} \end{vmatrix}$$

having the form of a bordered Gram determinant.

Let us make sure that this solution satisfies the basic relation as well. To do so, let us, just as in the previous case, present $\Gamma^1_{a_1\ldots a_r;b_1\ldots b_r}(w_2)$ in the form of a product of two determinants, each being identically equal to zero:

$$\Gamma^1_{a_1\ldots a_r;b_1\ldots b_r}(w_2) = \begin{vmatrix} 0 & 1 & \ldots & 1 \\ -1 & w_2(a_1,b_1) & \ldots & w_2(a_1,b_r) \\ \ldots & \ldots & \ldots & \ldots \\ -1 & w_2(a_r,b_1) & \ldots & w_2(a_r,b_r) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 & \ldots & 0 & 0 \\ x^{r-1}(a_1) & 1 & x^1(a_1) & \ldots & x^{r-1}(a_1) & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ x^{r-1}(a_r) & 1 & x^1(a_r) & \ldots & x^{r-1}(a_r) & 0 \end{vmatrix}$$

$$\times \begin{vmatrix} 0 & 1 & \ldots & 1 \\ -1 & \xi_{r-1}(b_1) & \ldots & \xi_{r-1}(b_r) \\ 0 & \xi_1(b_1) & \ldots & \xi_1(b_r) \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \xi_{r-2}(b_1) & \ldots & \xi_{r-2}(b_r) \\ 0 & 0 & \ldots & 0 \end{vmatrix} \equiv 0.$$

As for a proof of the uniqueness of these two solutions, this extremely difficult problem was successfully solved for an arbitrary rank by two different methods: by G.G. Mikhailichenko using a *functional* method [10, 11, 12] and by V.Kh. Lev using a *parametric* method [27].

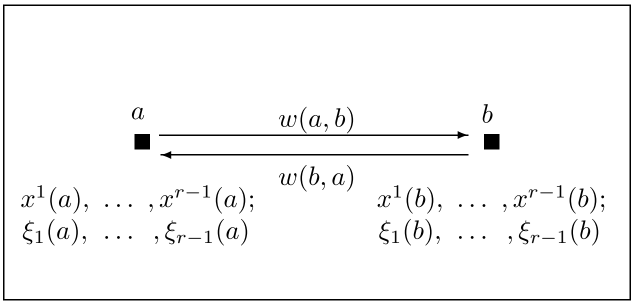

Figure 2: Contra- and covariant coordinates of two physical objects a and b in both metageometries.

4. Two meta-geometries with an asymmetric metric

We have seen that the rank r physical structures theory on a single set $\mathcal{A}$ directly implies the existence of two unusual $2(r-1)$-dimensional metaspaces with the asymmetric metrics

$$w_1(a,b) \neq w_1(b,a) \quad \text{and} \quad w_2(a,b) \neq w_2(b,a)$$

Each physical object a is then put into correspondence to $2(r-1)$ independent coordinates:

$r-1$ contravariant ones, $x^1(a),\dots,x^{r-1}(a)$ and
$r-1$ covariant ones, $\xi_1(a),\dots,\xi_{r-1}(a)$.

It can be noticed that the asymmetric representers $w_1(a,b)$ and $w_2(a,b)$, which play the role of asymmetric metrics, are in a certain sense *degenerate*:

the representers $w_1(a,b)$ and $w_2(a,b)$ do not depend on the coordinates $x^1(b),\ldots,x^{r-1}(b)$ and $\xi_1(a)$, $\ldots,\xi_{r-1}(a)$,

while the representers $w_1(b,a)$ and $w_2(b,a)$ do not depend on the coordinates $x^1(a),\ldots,x^{r-1}(a)$ and $\xi_1(b),\ldots,\xi_{r-1}(b)$.

It should be stressed that the representers' *linear dependence* on the coordinates, emerging in both cases, is not postulated at the outset, is not put into the structure "by hand", but arises by itself, in the only possible way, from very general assumptions.

Moreover, the division of the independent coordinates into two groups, the co- and contravariant ones, arises on the earliest metalevel, thus anticipating the emergence of dually conjugated vectors on the level of linear spaces.

5. Physical foundations of linear algebra

Let us return to the first solution A, creating the multiplicative metric $w_1(a,b)$, and impose the additional requirement of *symmetry*:

$$\forall\ a,b\in\mathcal{A} \qquad w_1(a,b)=w_1(b,a)$$

We have in this case

$$x^1(a)\xi_1(b)+\ldots+x^{r-1}(a)\xi_{r-1}(b) = x^1(b)\xi_1(a)+\ldots+x^{r-1}\xi_{r-1}(a) \qquad (3)$$

where $r-1=n$ is the dimension of the new emerging linear space.

As shown by G. Mikhailichenko [24], to make Eq. (3) an identity, valid for any a and b, it is necessary that there be a *linear dependence* between the co- and contravariant coordinates, i.e.,

$$\xi_\mu(b)=g_{\mu\nu}x^\nu(b)$$

where $\mu,\nu=1,2,\ldots,n$ and

$$g_{\mu\nu}=g_{\nu\mu}$$

is an arbitrary symmetric non-degenerate matrix of numbers, the global *metric tensor*.

Thus the representer $w_1(a,b)$ is written in the form

$$w_1(a,b)=g_{\mu\nu}x^\mu(a)x^\nu(b),$$

or, after the metric tensor diagonalization,

$$w_1(a,b)=e_1x^1(a)x^1(b)+\ldots+e_nx^n(a)x^n(b),$$

where $e_\mu=\pm1$ is the sign function.

Note that the symmetric metric tensor $g_{\mu\nu}$ comes into existence in a unique way, as a result of the symmetrization of the representer $w_1(a,b)$.

Putting $e_\mu=1$ for all $\mu=1,2,\ldots,n$, we obtain the well-known scalar product of two vectors

$$\begin{aligned}\vec a &= (x^1(a),\ldots,x^n(a),\\ \vec b &= (x^1(b),\ldots,x^n(b),\\ w_1(\vec a,\vec b) &= (\vec a,\vec b)=x^1(\vec a)x^1(\vec b)+,\ldots,x^n(\vec a)x^n(\vec b).\end{aligned}$$

6. Physical foundations of the constant curvature space geometries

Consider the symmetric multiplicative metric

$$w_1(a,b)=x^1(a)x^1(b)+\ldots+x^{r-1}(a)x^{r-1}(b) \qquad (4)$$

and impose the additional requirement of *reflexivity*

$$\forall\ a\in\mathcal{A} \qquad w_1(a,b)=eR^2$$

where $e=\pm1$ is the sign function and R^2 is an arbitrary positive constant.

We have in this case

$$(x^1(a))^2+\ldots+(x^{r-2}(a))^2+(x^{r-1}(a))^2=eR^2,$$

whence

$$x^{r-1}(a)=\sqrt{eR^2-(x^1(a))^2-\ldots-(x^{r-2}(a))^2}.$$

Substituting the resulting expression for $x^{r-1}(a)$ into (4), we find the final expression for $w_1(a,b)$:

if $e=1$, then

$$\begin{aligned}w_1^+(a,b) &= \sqrt{R^2-(x^1(a))^2-\ldots-(x^{r-2}(a))^2}\\ &\times\sqrt{R^2-(x^1(b))^2-\ldots-(x^{r-2}(b))^2}\\ &+x^1(a)x^1(b)+\ldots+x^{r-2}(a)x^{r-2}(b); \qquad (5)\end{aligned}$$

if $e=-1$, then

$$\begin{aligned}-w_1^-(a,b) &= \sqrt{R^2+(x^1(a))^2+\ldots+(x^{r-2}(a))^2}\\ &\times\sqrt{R^2+(x^1(b))^2+\ldots+(x^{r-2}(b))^2}\\ &-x^1(a)x^1(b)-\ldots-x^{r-2}(a)x^{r-2}(b). \quad (6)\end{aligned}$$

The expression (5) is the metric of an $(n{=}r{-}2)$-dimensional space of constant *positive* curvature (a global Riemann space) in a Cartesian coordinate frame.

As for the expression (6), it is the metric of an $(n=r-2)$-dimensional space of constant *negative* curvature (a global Lobachevsky space) in a Cartesian coordinate frame.

Let us introduce, instead of the representer $w_1(a,b)$, a new physical quantity, the distance $l(a,b)$ between the objects a and b.

Then, in the case of the global Riemann and Lobachevsky spaces, the fundamental relations between the distances $l^+(a,b)$ and $l^-(a,b)$, will have the forms, respectively,

$$\Phi^+\big(\begin{matrix} l^+(a_1,b_1), & \ldots & l^+(a_1,b_r),\\ \ldots & \ldots & \ldots\\ l^+(a_r,b_1) & \ldots & l^+(a_r,b_r)\end{matrix}\big) = \begin{vmatrix}\cos\frac{l^+(a_1,b_1)}{R} & \ldots & \cos\frac{l^+(a_1,b_r)}{R}\\ \ldots & \ldots & \ldots\\ \cos\frac{l^+(a_r,b_1)}{R} & \ldots & \cos\frac{l^+(a_r,b_r)}{R}\end{vmatrix}\equiv 0$$

$$\Phi^-\big(\begin{matrix} l^-(a_1,b_1), & \ldots & l^-(a_1,b_r),\\ \ldots & \ldots & \ldots\\ l^-(a_r,b_1) & \ldots & l^-(a_r,b_r)\end{matrix}\big) = \begin{vmatrix}\mathrm{Ch}\frac{l^-(a_1,b_1)}{R} & \ldots & \mathrm{Ch}\frac{l^-(a_1,b_r)}{R}\\ \ldots & \ldots & \ldots\\ \mathrm{Ch}\frac{l^-(a_r,b_1)}{R} & \ldots & \mathrm{Ch}\frac{l^-(a_r,b_r)}{R}\end{vmatrix}\equiv 0.$$

Thus, starting from a $2(r-1)$-dimensional metaspace with an asymmetric representer $w_1(a,b)$ and imposing the additional condition of **symmetry**

$$\forall\ a,b \in \mathcal{A} \qquad w_1(a,b) = w_1(b,a)$$

we obtain the well-known structure of an $(r-1)$-dimensional linear space.

The further requirement of **reflexivity**

$$\forall\ a \in \mathcal{A} \qquad w_1(a,a) = eR^2$$

creates the structures of $(r-2)$-dimensional spaces of constant positive $(e=1)$ and negative $(e=-1)$ curvature (respectively, the Riemann and Lobachevsky spaces) [13, 22, 23].

On the other hand, if one imposes the additional condition of **antisymmetry**

$$\forall\ a,b \in \mathcal{A} \qquad w_1(a,b) = -w_1(b,a),$$

then, for even r, the structure of even-dimensional symplectic spaces is obtained.

7. Physical foundations of Euclidean geometry

Let us now pass over to considering the second $2(r-1)$-dimensional metaspace with the asymmetric representer $w_2(a,b)$.

Impose the additional condition of **symmetry** on the representer

$$\forall\ a,b \in \mathcal{A} \qquad w_2(a,b) = w_2(b,a).$$

In this case

$$\begin{aligned} x^1(a)\xi_1(b) + \ldots + x^{r-2}(a)\xi_{r-2}(b) + x^{r-1}(a) + \xi_{r-1}(b) \\ = x^1(b)\xi_1(a) + \ldots \\ + x^{r-2}(b)\xi_{r-2}(a) + x^{r-1}(b) + \xi_{r-1}(a). \end{aligned} \qquad (7)$$

As shown by G. Mikhailichenko [24], to make Eq. (7) an identity, valid for any a and b, it is necessary that there be a *linear dependence* between the co- and contravariant coordinates, i.e.,

$$\xi_\mu(b) = g_{\mu\nu}x^\nu(b), \qquad \mu,\nu = 1,2,\ldots,r-2$$

where $g_{\mu\nu} = g_{\nu\mu}$ is a symmetric metric tensor and

$$\xi_{r-1}(b) = x^{r-1}(b).$$

The symmetric representer $w_2(a,b)$ is thus written in the form

$$w_2(a,b) = g_{\mu\nu}x^\mu(a)x^\nu(b) + x^{r-1}(a) + x^{r-1}(b). \qquad (8)$$

Impose one more requirement, that of **reflexivity**, on the symmetric representer (8), i.e., require

$$\forall\ a \in \mathcal{A} \qquad w_2(a,a) = C,$$

with an arbitrary constant C.

Then

$$g_{\mu\nu}x^\mu(a)x^\nu(a) + 2x^{r-1}(a) = C,$$

whence

$$x^{r-1}(a) = \frac{C}{2} - \frac{1}{2}g_{\mu\nu}x^\mu(a)x^\nu(a)$$

and

$$x^{r-1}(b) = \frac{C}{2} - \frac{1}{2}g_{\mu\nu}x^\mu(b)x^\nu(b).$$

Substituting the resulting expression into (8), we obtain:

$$w_2(a,b) = C - \frac{1}{2}l^2(a,b)$$

where

$$l^2(a,b) = g_{\mu\nu}[x^\mu(a) - x^\mu(b)]\cdot[x^\nu(a) - x^\nu(b)],$$

$$\mu,\nu = 1,2,\ldots,n = r-2,$$

or, after the metric tensor diagonalization,

$$l^2(a,b) = e_1[x^1(a) - x^1(b)]^2 + \ldots + e_n[x^n(a) - x^n(b)]^2$$

where $n = r-2$ is the dimension of the pseudo-Euclidean space with the signature $(e_1,\ldots,e_n)$.

Thus, the theory of physical structures of rank r on a single set $\mathcal{A}$ implies the existence of two and only two $2(r-1)$-dimensional metaspaces with the asymmetric metrics $w_1(a,b)$ and $w_2(a,b)$.

Starting from the $2(r-1)$-dimensional metaspace with the metric $w_1(a,b)$, we obtain the $(r-1)$-dimensional linear algebra, the $r-2$-dimensional Riemann and Lobachevsky constant curvature spaces and, for even r, the even-dimensional symplectic geometry.

Starting from the $2(r-1)$-dimensional metaspace with the metric $w_2(a,b)$ and imposing the additional **symmetry** condition, we obtain a new $(r-)1$-dimensional hyperspace with the symmetric metric

$${}^s w_2(a,b) = x_a^1x_b^1 + \ldots + x_a^{r-2}x_b^{r-2} + x_a^{r-1} + x_b^{r-1}.$$

The further **reflexivity** requirement creates the well-known $r-2 = n$-dimensional pseudo-Euclidean space with the metric

$$\begin{aligned} -2\cdot{}^{so}w_2(a,b) &= l_{ab}^2 \\ &= e_1(x_a^1 - x_b^1)^2 + \ldots + e_n(x_a^n - x_b^n)^2. \end{aligned}$$

Let us illustrate the transition from the $2(r-1)$-dimensional metaspace to the $(r-2)$-dimensional Euclidean space by the example of physical structures of ranks 2, 3, 4 and 5:

Rank $r = 2$ physical structure:

1a. Asymmetric metric

$$w_{ab} = s_a + \sigma_b$$

of the 2-dimensional metaspace;

1b. Symmetric metric

$$^s w_{ab} = s_a + s_b$$

of the 1-dimensional hyperspace;

1c. Cartesian metric

$$-2^{so} w_{ab} = l^2_{ab} = 0$$

of the zero-dimensional Euclidean space.

Rank $r = 3$ physical structure:

2a. Asymmetric metric

$$w_{ab} = x_a \xi_b + s_a + \sigma_b$$

of the 4-dimensional metaspace;

2b. Symmetric metric

$$^s w_{ab} = x_a x_b + s_a + s_b$$

of the 2-dimensional hyperspace;

2c. Cartesian metric

$$-2^{so} w_{ab} = l^2_{ab} = (x_a - x_b)^2$$

of the 1-dimensional Euclidean space.

Rank $r = 4$ physical structure:

3a. Asymmetric metric

$$w_{ab} = x_a \xi_b + y_a \eta_b + s_a + \sigma_b$$

of the 6-dimensional metaspace;

3b. Symmetric metric

$$^s w_{ab} = x_a x_b + y_a y_b + s_a + s_b$$

of the 3-dimensional hyperspace;

3c. Cartesian metric

$$-2^{so} w_{ab} = l^2_{ab} = (x_a - x_b)^2 + (y_a - y_b)^2$$

of the 2-dimensional Euclidean space.

Rank $r = 5$ physical structure:

4a. Asymmetric metric

$$w_{ab} = x_a \xi_b + y_a \eta_b + z_a \zeta_b + s_a + \sigma_b$$

of the 8-dimensional metaspace;

4b. Symmetric metric

$$^s w_{ab} = x_a x_b + y_a y_b + z_a z_b + s_a + s_b$$

of the 4-dimensional hyperspace;

4c. Cartesian metric

$$-2^{so} w_{ab} = l^2_{ab} = (x_a - x_b)^2 + (y_a - y_b)^2 + (z_a - z_b)^2$$

of the 3-dimensional Euclidean space.

8. Conclusion

As known, a unified viewpoint on different global geometries was for the first time formulated by Felix Klein in 1872 and acquired the name of the *Erlangen programme*. Its essence was that choosing different transformation groups, one can obtain different global geometries. However, as found out by G. Mikhailichenko [11], not even nearly each group creates a consistent, i.e., non-degenerate geometry. Thus there again occurs a problem of selecting a small number of consistent geometries from the enormous set of geometries with degenerate metrics created by arbitrarily chosen groups.

The proposed *Novosibirsk programme* [22], whose basis is the physical structures theory, contains no arbitrariness and there is no necessity of inserting something “by hand” from outside. It is sufficient to fix the dimension, or, which is the same, to specify the rank of a physical structure, for obtaining a finite set of concrete geometries whose metrics come into existence by themselves, just from the requirement of the **phenomenological symmetry** underlying the physical structures theory, the requirement of **symmetry or antisymmetry under permutation of a pair of points** and the **reflexivity** requirement.

It should be noted that the geometries emerging in this way turn out to be the most consistent, exerting a clear influence upon the whole modern theoretical natural science.

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