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BINARY GEOMETROPHYSICS: SPACE-TIME, GRAVITATION

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A new conception for building a theory of space-time and physical interactions, called binary geometrophysics, is suggested. It rests on the ideas of (i) the Fokker-Feynman direct particle interaction theory, (ii) Kaluza-Klein type multidimensional geometric models of physical interactions and (iii) Kulakov's theory of binary physical structures. We describe in its frames a transition from binary systems of complex relations (a sort of binary geometries) to conventional (unary) geometries with symmetry groups. We show that the 4-dimensional Minkowski geometry with pairwise metric relations corresponds to binary systems of complex relations of rank (3,3). Higher ranks of binary multidimensional systems lead to multi-point geometric constructions. A transition from rank (4,4) binary systems of complex relations to multidimensional Kaluza-Klein theories and, in particular, to Einstein's general relativity is demonstrated.

1. Introduction

A foundation of theoretical physics is the theory of space-time. All main achievements of physics of the 20th century: special relativity, general relativity and quantum theory, *are connected with changing the views of space-time properties*. Nowadays the theoretical physicists become more and more convinced that the real space-time geometry is physics, and the foundations of physics should be described in terms of generalized space-time geometry. A further progress in fundamental theoretical physics should be expected on the path of one more revision of the outlook on the essence of physical space-time.

Modern physics is built on the basis of a ready (flat or curved) space-time as a receptacle of everything that exists. These views are common since the epoch of Descartes and Newton. The author is convinced that the further development of physics and geometry should be connected with a transition to a *relational treatment of space-time*, where the latter is understood as a system of relations between material objects. In such a view, there is no space-time without matter. This approach is commonly connected with the names of G. Leibniz and E. Mach [1]. It is alternative to the substantial view on the nature of space and time, adhered to by W. Clifford and J. Wheeler.

A suitable basis for developing the relational space-time conception is the *theory of binary physical structures* built by Yu.I. Kulakov in order to re-formulate a number of laws of general physics [2]. This theory postulates the existence of two sets of elements and their mutual relations, satisfying certain algebraic conditions. In Kulakov's theory these relations are real numbers put into correspondence to the elements for a single set or two different ones.

We believe that a new physical picture of the world should rest on a system of elementary notions borrowed from microworld physics, giving rise to both the classical space-time notions and a theory of the known fundamental physical interactions. To build such a theory, named *binary geometrophysics* (BGP) [3], we made use of complexified binary physical structures of symmetric ranks (r,r). Only in this case it proves to be possible to reflect the physical properties of the microworld. Simplified mathematical models of concrete ranks in this theory were named *binary systems of complex relations* (BISCR).

In BISCR of the first non-degenerate rank (3,3) one can build a *prototype of the 4-dimensional classical Minkowski space-time*, and the 3-dimensional hyperbolic (Lobachevsky) geometry, interpreted in BGP as the *momentum space* of a selected class of free particles. It is suggested to treat them as idealized (non-interacting, first generation) massive leptons.

The present work shows that in order to obtain a realistic theory of interacting particles it is necessary to employ higher-rank BISCR, i.e., *binary multidimensionality*, representing a prototype for the well-known (unary) multidimensional Kaluza-Klein theories. The ideas of multidimensionality form the second block of basic principles for building BGP.

A theory of physical interactions should be built in BGP in the spirit of *direct particle interaction theory* (DPIT) of Fokker-Feynman type, with a difference that both the prototype of action and the space-time relations are now described by notions of the same type, corresponding to inter-particle relations. The DPIT ideas form the third block of BGP principles.

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This paper demonstrates a transition from BISCR(4,4) with the corresponding flat 3-point geometry to *multi-dimensional curved 2-point geometry* used in Kaluza-Klein theories. The curved space-time of general relativity is then to be understood as a 4-dimensional section of a multidimensional 2-point geometry, obtained from BISCR(4,4) by its reduction to a 4-dimensional theory.

2. Basic concepts of binary geometrophysics

The theory of binary systems of relations (binary structures) has been presented in a number of our papers [3,4]. We will recall the most necessary concepts. The theory starts from a law Φ for relations $u_{i\alpha}$ between the elements of two sets $i \in \mathcal{M}$ and $\alpha \in \mathcal{N}$. In the first set $\mathcal{M}$ the elements are designated by Latin indices, in the second set by Greek ones. The rank (4,4) means that the law is written for 4 arbitrary elements of the set $\mathcal{M}$ and 4 arbitrary elements of the set $\mathcal{N}$. By the general theory, the law of BISCR(4.4) for the elements i, k, j, s and α, β, γ, δ is written in the form

$$\Phi(u_{i\alpha},\ldots) = \begin{vmatrix} u_{i\alpha} & u_{i\beta} & u_{i\gamma} & u_{i\delta} \\ u_{k\alpha} & u_{k\beta} & u_{k\gamma} & u_{k\delta} \\ u_{j\alpha} & u_{j\beta} & u_{j\gamma} & u_{j\delta} \\ u_{s\alpha} & u_{s\beta} & u_{s\gamma} & u_{s\delta} \end{vmatrix} = 0. \qquad (1)$$

It is easily shown that this law is satisfied identically if each element is characterized by three complex numbers ($i \to i^1,\ i^2,\ i^3;\ \alpha \to \alpha^1,\ \alpha^2,\ \alpha^3$), so that a binary relation is expressed in the form

$$u_{i\alpha} = i^1\alpha^1 + i^2\alpha^2 + i^3\alpha^3. \qquad (2)$$

This is actually a scalar product of two vectors in a 3-dimensional complex space.

The BISCR(4,4) theory may be understood as a sort of multidimensional generalization of the rank (3,3) BISCR, responsible for the observed classical 4-dimensionality [5]. Recall that the BISCR(3,3) law is written in a form similar to (1) but for two triads of elements from different sets,

$$\Phi(u_{i\alpha},\ldots) = \begin{vmatrix} u_{i\alpha} & u_{i\beta} & u_{i\gamma} \\ u_{k\alpha} & u_{k\beta} & u_{k\gamma} \\ u_{j\alpha} & u_{j\beta} & u_{j\gamma} \end{vmatrix} = 0, \qquad (3)$$

when the elements are characterized by only two complex parameters and their binary relation has the form

$$u_{i\alpha} = i^1\alpha^1 + i^2\alpha^2. \qquad (4)$$

In such a theory, of key significance is the so-called fundamental 2×2 relation

$$\begin{bmatrix} \alpha\ \beta \\ i\ \ k \end{bmatrix} \equiv \begin{vmatrix} u_{i\alpha} & u_{i\beta} \\ u_{k\alpha} & u_{k\beta} \end{vmatrix} = \begin{vmatrix} i^1 & k^1 \\ i^2 & k^2 \end{vmatrix} \times \begin{vmatrix} \alpha^1 & \beta^1 \\ \alpha^2 & \beta^2 \end{vmatrix}, \qquad (5)$$

put into correspondence to two pairs of unlike elements. Linear transformations of elements

$$i'^s = C^s_r i^r; \quad \alpha'^s = \overset{*}{C}{}^s_r\, \alpha^r, \quad (s,r = 1,2) \qquad (6)$$

with complex coefficients C^s_r, leaving invariant separate determinants on the right-hand side of (5), form the $SL(2,C)$ group, while the relation (5) itself may be presented in terms of the quadratic form

$$\begin{bmatrix} \alpha\ \beta \\ i\ \ k \end{bmatrix} = p_0^2 - p_1^2 - p_2^2 - p_3^2 = \eta_{\mu\nu}p^\mu p^\nu, \qquad (7)$$

where $\eta_{\mu\nu}$ is the metric tensor of the 4-dimensional Minkowski space-time, p_μ are the components of a 4-vector formed by the parameters of the two pairs of elements: i, k, α, β. If the elements i, α and k, β are described by complex-conjugate parameters, then the vector p_μ is real.

Similar considerations for BISCR(4,4) single out the $SL(3,C)$ transformation group. The parameters of elements allow one to build a real 9-dimensional vector. Singling out of the $SL(2,C)$ transformation group corresponds to a reduction of the BISCR(4,4) theory to the BISCR(3,3) theory, just as multidimensional Kaluza-Klein-like models are reduced to 4-dimensional general relativity with additional fields of geometric origin. As is well-known, the latter are described by additional components of the multidimensional metric tensor $G_{5\mu}$, $G_{6\mu}$ etc. In the present context one should act in a similar way: build the 4-dimensional vectors from parameters with the indices 1 and 2 and, using the additional parameters, define the particle charges [4,6].

It is assumed in binary geometrophysics that the BISCR(3,3) model describes idealized (non-interacting) leptons: two pairs of elements from different sets describe massive leptons (electrons and positrons), while the single pair of unlike elements describes the neutrino. Realistic, or, in other words, interacting (in the electroweak manner) leptons are described by the same elements but taken from the reduced BISCR(4,4) theory (see [4]).

3. Binary systems of complex relations and unary symmetric geometries

The experience of work in Einstein's general relativity, Kaluza-Klein theories and quantum theory shows that there is an important part of each of these theories that describes a transition from primary notions to physically observed or interpreted quantities. In general relativity these are the methods of describing frames of reference, in Kaluza-Klein theories the methods of reducing multidimensional relations and quantities to 4-dimensional ones, and in quantum theory it is the transition to Hermitian operators and their eigenvalues. A similar part is contained in binary geometrophysics. It consists in a transition from BISCR to unary systems of real relations (USRR), leading to the familiar physical and geometric meaning of the concepts used.

This is achieved by uniting pairs or greater numbers of elements from the two different sets of BISCR to form a certain new structure able to play the role of an element of a unary system of real relations. The USRR relations are built from binary relations of the original BISCR. As a rule, one "glues" the elements of the two sets with complex-conjugate parameters. The general theory of USRR on a single set of elements, under the name of physical structures theory, was built in the papers by Yu.I. Kulakov, G.G. Mikhailichenko [7] and V.Kh. Lev [8]. It was shown that such structures correspond to the known types of geometries with symmetry groups. Laws of certain ranks, like the BISCR laws presented in (1) and (2), are written for such structures. Thus, it turns out that the 4-dimensional Minkowski space-time geometry is described by a rank 6 USRR. Its law is written as the vanishing of the Cayley-Menger determinant for six points:

$$\Phi = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & a_{ik} & a_{ij} & a_{is} & a_{il} & a_{im} \\ 1 & a_{ki} & 0 & a_{kj} & a_{ks} & a_{kl} & a_{km} \\ 1 & a_{ji} & a_{jk} & 0 & a_{js} & a_{jl} & a_{jm} \\ 1 & a_{si} & a_{sk} & a_{sj} & 0 & a_{sl} & a_{sm} \\ 1 & a_{li} & a_{lk} & a_{lj} & a_{ls} & 0 & a_{lm} \\ 1 & a_{mi} & a_{mk} & a_{mj} & a_{ms} & a_{ml} & 0 \end{vmatrix} = 0, \quad (8)$$

with pairwise relations presented in the form

$$a_{ik} \equiv s^2_{ik} = (x^0_i - x^0_k)^2 - \sum_{l=1}^{3}(x^i_i - x^l_k)^2. \quad (9)$$

Here x^μ_i and x^μ_k are the coordinates of points (events) in the 4-dimensional space-time. Refs. [2,3] and a number of others contain the laws for other possible geometries. It was shown, in particular, that, for rank 5, there are 10 and only 10 possible 3-dimensional geometries. The geometry dimension n and the real structure rank r are connected by the relation $n = r - 2$. Among them there are the Euclidean, pseudo-Euclidean and Lobachevsky geometries, that of Riemann (of constant positive curvature), the symplectic one and some others.

It turns out that the 4-dimensional Minkowski space-time can be obtained from BISCR(3,3). Besides, one can simultaneously obtain the 3-dimensional Lobachevsky geometry [9], interpreted in BGP as the momentum space for massive leptons.

Just as was done for BISCR(3,3), one can pass to USRR from BISCR of rank (4,4) and higher. It then turns out that the resulting geometric constructions have not only a higher dimension (in a certain generalized sense), but also other definitions of measure. *The conventional (quadratic) metric relations turn out to be inherent to only the rank* $(3,3)$ *BISCR*. The rank (4,4) BISCR yields *three-point geometries*, i.e., such that a metric is specified for three points. BISCR(5,5) models lead to 4-point geometries, etc. It should be noted that, apart from BGP, multi-point geometries were considered by V.Ya. Skorobogat'ko [10]. One can write down for such multi-point geometries some laws in the spirit of Kulakov's theory of real physical structures. However, unlike the conventional geometries with quadratically defined measures, such laws are written using, instead of square, cubic and other spatial determinants. There is a sufficiently well-defined theory of such determinants [11] but it has practically not been used in theoretical physics. In our papers such laws were written and a specific form of metric, generalizing the Minkowski space-time metric, has been introduced [12].

4. The basic 4×4 relation and averaging over reference elements

So far we discussed unary geometries possessing symmetries: the Minkowski space-time, the Lobachevsky geometry and multi-point analogues of these and other geometries with groups of motions. However, *in binary geometrophysics a curved, Riemannian type space-time geometry can be obtained as well.* This is carried out using some rather natural principles.

Principle 1. Since Riemannian geometry, being a basis for general relativity and the Kaluza-Klein theories, is a unary geometry with pairwise real relations, whereas BISCR(4,4) yields 3-point geometries, *a transition from a three-point metric to a two-point one is performed by summing the three-point relations with two selected "points" over all third "points".* The latter can be symbolically presented as follows:

$$a(1,1') = \sum_2 a(1,1',2), \quad (10)$$

where the left-hand side is the pairwise relation between the selected "points" 1 and 1′ and the right-hand side is a sum of triple relations over all third points designated symbolically by the digit 2 (as the second particles).

Principle 2. We will choose the original triple relations in the form of expressions which are, first, built symmetrically from the parameters of the two tetrads of elements describing two massive particles (a pair of leptons in the BISCR(4,4) model) and, second, invariant with respect to the $SL(3,C)$ transformation group, characteristic of the BISCR(4,4) model. Such properties are possessed by the *basic* 4×4 *relation*, written in terms of the bordered determinant of pairwise relations

$$\left\{ \begin{matrix} \alpha\beta\gamma\delta \\ ikjs \end{matrix} \right\} = - \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & u_{i\alpha} & u_{i\beta} & u_{i\gamma} & u_{i\delta} \\ 1 & u_{k\alpha} & u_{k\beta} & u_{k\gamma} & u_{k\delta} \\ 1 & u_{j\alpha} & u_{j\beta} & u_{j\gamma} & u_{j\delta} \\ 1 & u_{s\alpha} & u_{s\beta} & u_{s\gamma} & u_{s\delta} \end{vmatrix}, \quad (11)$$

where the elements i, k, α and β correspond to a selected particle (1), while j, s, γ and δ define the second particle (2). Evidently, this expression is written in the form of a set of 16 fundamental 3×3 relations, invariant with respect to the group $SL(3,C)$.

Principle 3. *To pass to classical concepts it is necessary to carry out an averaging over the full set of basic elements forming a classical macro-instrument* (a classical observer). It should be stressed that until now the parameters of elements had only the meaning of relations to elements of a reference set. That might be just a separate particle.

The averaging over the full set of basic elements includes the following main procedures:

(a) Summing over the third elements of the basis, interpretable in the spirit of Mach's principle, since the third parameters of elements are treated as certain effective relations to all particles of the world. That means in practice that an $SL(2,C)$ transformation subgroup, concerning solely the parameters labelled 1 and 2, is singled out. The third parameter remains invariant. As a result, the basic 4×4 relation (11) is represented as a sum of 36 Lorentz-invariant summands of the form $\begin{bmatrix}\alpha\gamma\\ ij\end{bmatrix}\cdot\begin{pmatrix}\beta\delta\\ ks\end{pmatrix}$; here square brackets enclose the fundamental 2×2 relation (5), while parantheses denote the combination of third parameters

$$\begin{pmatrix}\beta\delta\\ ks\end{pmatrix} = (k^3-s^3)(\delta^3-\beta^3). \tag{12}$$

(b) Summing over all different systems of relations characterized by different non-degenerate parameter values (with the indices 1 and 2). This is expressed, in particular, in integration over exchange momenta of interacting particles ("intermediate boson" momenta).

Principle 4. To pass over to classical concepts one should postulate that *both the selected "particle" and the surrounding ones (2) are macro-objects*. That means, in particular, averaging over polarizations of constituent particles. The particle momenta are determined by parameters labelled 1 and 2 in an ambiguous way, up to transformations on a 3-dimensional hypersphere. The averaging over polarizations means integration over this hypersphere. After such an averaging, survive only 6 "diagonal" summands from a total of the above 36 Lorentz-invariant summands of the basic 4×4 relation. The surviving expression is

$$\begin{aligned} a(1,\ 1'\ ,2\ ,2') &\equiv L(e_1,e_2)\\ &= \begin{bmatrix}\alpha\beta\\ ik\end{bmatrix}\begin{pmatrix}\gamma\delta\\ js\end{pmatrix} + \begin{bmatrix}\alpha\gamma\\ ij\end{bmatrix}\begin{pmatrix}\beta\gamma\\ ks\end{pmatrix} + \begin{bmatrix}\alpha\delta\\ is\end{bmatrix}\begin{pmatrix}\beta\gamma\\ kj\end{pmatrix}\\ &+ \begin{bmatrix}\beta\gamma\\ kj\end{bmatrix}\begin{pmatrix}\alpha\delta\\ is\end{pmatrix} + \begin{bmatrix}\beta\delta\\ ks\end{bmatrix}\begin{pmatrix}\alpha\gamma\\ ij\end{pmatrix} + \begin{bmatrix}\gamma\delta\\ js\end{bmatrix}\begin{pmatrix}\alpha\beta\\ ik\end{pmatrix}. \end{aligned} \tag{13}$$

This expression was used in Ref.[4] as an algebraic analogue of the electroweak interaction Lagrangian for two massive leptons.

Besides, a transition to macro-objects means that the summands containing pseudo-vector factors (corresponding in standard theory to an interaction via Z bosons), are neglected. The vector summands are interpreted as 4-dimensional particle momenta by the definition given in [3].

5. Basic principles of transition to multidimensional Kaluza-Klein theory

Principle 5. For each of the particles *two combinations of additional parameters of their constituent elements have the physical meaning of the 5th and 6th components of the multidimensional momentum*:

$$\begin{aligned} &i^3+k^3 \equiv C_{1L}+C_{1R}\to p^5_{(1)};\\ &i^3-k^3 \equiv C_{2L}-C_{2R}\to p^6_{(2)};\\ &j^3+s^3 \equiv C_{2L}+C_{2R}\to p^5_{(2)};\\ &j^3-s^3 \equiv C_{2L}-C_{2R}\to p^6_{(2)}. \end{aligned} \tag{14}$$

Similar expressions are valid for conjugate parameters.

This principle means actually a transition to a degenerate BISCR of rank (4,4;a) [3], such that the third parameters are represented in the form

$$i^3 = \frac{1}{\sqrt{\varepsilon}}\mathrm{e}^{\varepsilon i_0};\quad \alpha^3 = \frac{1}{\sqrt{\varepsilon}}\mathrm{e}^{\varepsilon\alpha_0}, \tag{15}$$

where i_0 and α_0 are new parameters with the dimension of momentum. Recall that in BISCR(4,4;a) theory binary relations are presented in the form

$$a_{i\alpha} = i^1\alpha^1 + i^2\alpha^2 + i_0 + \alpha_0. \tag{16}$$

If the parameters with the index 3 are replaced by those with the index 0, the arrows in (14) turn into equality signs.

Principle 6. *For interacting particles, the initial and final state parameters are no longer connected by the complex conjugation condition.* This condition is generalized to

$$(\bar e_{1L}\gamma^\mu e_{1L}) + (\bar e_{1R}\gamma^\mu e_{1R}) = (p'^\mu_{(1)} - k^\mu)\mathrm{e}^{i(\phi'_1-\phi_1)} \tag{17}$$

where the left-hand side is the complex expression built for an interacting particle by the conventional rules; in the right-hand side, $p'^\mu_{(1)}$ is the final momentum of the first particle, k^μ is the transition momentum from the second particle, and the real quantity ϕ describes the initial and final state phases.

Principle 7. *The phases of the exponential summands introduced in (17) are expressed in terms of the 4-momenta and the classical space-time coordinates.* For final states the factor is

$$\alpha^s,\beta^s \sim \mathrm{e}^{i\phi'_1} = \exp[\tfrac{i}{\hbar}p'_{(1)\mu}x'^\mu_{(1)}] \tag{18}$$

where $s=1,2$; $x'^\mu_{(1)}$ are the final values of the particle coordinates. For initial states the factor is written in a similar way:

$$i^s,k^s \sim \mathrm{e}^{-i\phi_1} = \exp[-\tfrac{i}{\hbar}p_{(1)\mu}x^\mu_{(1)}], \tag{19}$$

where $p_{(1)\mu}$ and $x^{\mu}_{(1)}$ are the initial values of the particle momentum and coordinates.

Evidently, the product of the exponential summands in (17) may be presented in the form

$$\exp[\tfrac{i}{\hbar}(p'_{(1)\mu}x'^{\mu}_{(1)} - p_{(1)\mu}x^{\mu}_{(1)})] = \exp[\tfrac{i}{\hbar}dS_1]\cdot\exp[\tfrac{i}{\hbar}k_\mu x^{\mu}_{(1)}], \quad (20)$$

where we have put

$$dS_1 = p'_{(1)\mu}dx^{\mu}_{(1)}. \quad (21)$$

In what follows we will assume that dS_1 is small and expand the exponent in a series in dS_1, preserving the zero, first and second orders:

$$\exp[\tfrac{i}{\hbar}dS_1] \simeq 1 + \frac{i}{\hbar}dS_1 - \frac{1}{2\hbar^2}dS_1^2. \quad (22)$$

The quantity dS_1 may be presented as $dS_1 = mc\,ds_1$ where m is the particle mass and ds_1 is a displacement along its classical trajectory. Then the pairwise relation (10) acquires the form

$$a(1,1') = a_0(1,1') + \frac{imc}{\hbar}a_1(1,1')ds_1 - \frac{m^2c^2}{2\hbar^2}a_2(1,1')ds_1^2 + O(3). \quad (23)$$

Principle 8. We postulate that *the zero and first orders in the expression (23) vanish*:

$$a_0(1,1') = 0; \quad (24)$$
$$a_1(1,1') = 0, \quad (25)$$

i.e., the *pairwise relation* $a(1,1')$ *is in the principal approximation proportional to the squared 4-dimensional interval along the selected particle's "world line"*:

$$a(1,1') = -\frac{m^2c^2}{2\hbar^2}a_2(1,1')ds_1^2 + O(3). \quad (26)$$

This principle reduces the summed effective pairwise relation of the selected particle, built in BISCR(4,4), to a pairwise relation of the BISCR(3,3) theory.

Principle 9. We postulate that ds_1 is the particle displacement along some additional coordinate x^4, then the sum and the difference of (24) and (25) yield two expressions. One of them, multiplied by ds_1^2, is to be interpreted as a *squared null displacement in a 7-dimensional curved manifold*. We thus arrive at 7-optics, generalizing in a certain sense Rumer's 5-optics [13]. The three additional coordinates x^4, x^5, x^6 correspond to the classical action (displacement along the particle trajectory), the particle's electric charge (the momentum component p^5 means physically the electric charge, just as it does in the Kaluza-Klein theory) and a new parameter which characterized the interaction via an intermediate Z boson in the algebraic model of electroweak interactions.

Let us write down the 7-optics condition explicitly:

$$dI^2_{(7)} = \tilde{G}_{AB}dx^A dx^B = 0, \quad (27)$$

where the 7-dimensional metric has the form

$$\tilde{G}_{\mu\nu} = \frac{1}{2}\eta_{\mu\nu}\sum_{(2)}(p^6_{(2)})^2\exp[2\tfrac{i}{\hbar}k_\lambda x^\lambda_1]; \quad (28)$$

$$\tilde{G}_{\mu 4} \simeq \frac{1}{16}\sum_{(2)}p_{(2)\mu}[(p^5_{(2)})^2 + (p^6_{(2)})^2] \times \exp\{\tfrac{i}{\hbar}[dS_2 + k_\lambda(x^\lambda_1 + x^\lambda_2)]\}; \quad (29)$$

$$\tilde{G}_{\mu 5} \simeq \frac{1}{8}\sum_{(2)}p_{(2)\mu}p^5_{(2)} \times \exp\{\tfrac{i}{\hbar}[dS_2 + k_\lambda(x^\lambda_1 + x^\lambda_2)]\}; \quad (30)$$

$$\tilde{G}_{44} \simeq -\frac{1}{8}k_\mu\sum_{(2)}p^\mu_{(2)}[(p^5_{(2)})^2 + (p^6_{(2)})^2] \times \exp\{\tfrac{i}{\hbar}[dS_2 + k_\lambda(x^\lambda_1 + x^\lambda_2)]\}; \quad (31)$$

$$\tilde{G}_{45} = \frac{1}{8}k_\mu\sum_{(2)}p^\mu_{(2)}p^5_{(2)} \times \exp\{\tfrac{i}{\hbar}[dS_2 + k_\lambda(x^\lambda_1 + x^\lambda_2)]\}; \quad (32)$$

$$\tilde{G}_{55} = \tilde{G}_{66} = -\frac{1}{8}k_\mu\sum_{(2)}p^\mu_{(2)} \times \exp\{\tfrac{i}{\hbar}[dS_2 + k_\lambda(x^\lambda_1 + x^\lambda_2)]\};$$
$$\tilde{G}_{46} = \tilde{G}_{56} = 0. \quad (33)$$

Here the summation sign denotes summing both over particles forming the objects under consideration and over the macro-instrument relation systems.

6. The electromagnetic interaction

Let us single out in the metric (27) the terms describing the electromagnetic interaction:

$$dI^2_{(5)} = \tilde{G}_{AB}dx^A dx^B \neq 0, \quad (34)$$

where $A,B = 0,1,2,3,5$. Bearing in mind that the metric component $\tilde{G}_{5\mu}$, is, just as in the Kaluza-Klein theory, proportional to the electromagnetic vector potential, it is natural to put

$$\frac{1}{2}k_\mu\sum_{(2)}(p'^\mu_{(2)} + p^\mu_{(2)})\exp\{\tfrac{i}{\hbar}[dS_2 + k_\lambda(x^\lambda_1 + x^\lambda_2)]\} = \epsilon, \quad (35)$$

where $|\epsilon| \ll 1$. This corresponds to the Lorentz condition, well-known in electrodynamics. So one can write

$$\tilde{G}_{55} = -\frac{1}{16}(\epsilon - k_\mu k^\mu)\sum_{(2)}\exp\{\tfrac{i}{\hbar}[dS_2 + k_\lambda(x^\lambda_1 + x^\lambda_2)]\}. \quad (36)$$

As known from the modern formulation of the Kaluza-Klein theory in 5 dimensions [14], two procedures are necessary for identifying the multidimensional metric components with physical quantities: (1) a conformal transformation of the original metric and (2) a (1+4) splitting procedure. Let us choose the following *conformal factor*:

$$\mathcal{F} = \frac{1}{2}(\epsilon - k_\mu k^\mu)\exp\{\frac{2i}{\hbar}k_\lambda x^\lambda_1\}\sum_{(2)}(p^6_{(2)})^2, \quad (37)$$

so that $\tilde{G}_{AB} = \mathcal{F} G_{AB}$.

The (1+4) *splitting procedure* should be performed with the metric G_{AB}. Let us use to this end the monad method in a gauge similar to the chronometric gauge in 4-dimensional general relativity (see Ref.[15]). Using standard formulas, we obtain:

$$\breve{A}_\mu \sim \frac{G_{5\mu}}{\sqrt{G_{55}}} = \frac{1}{2\sqrt{\sum (p^6_{(2)})^2}} \times \sum_{(2)} p_{(2)\mu} p^5_{(2)} \frac{\exp\{\frac{i}{\hbar}[dS_2 + k_\lambda(x^\lambda_2 - x^\lambda_1)]\}}{\epsilon - k_\nu k^\nu}; \qquad (38)$$

$$\breve{g}_{\mu\nu} = \frac{1}{\epsilon - k_\beta k^\beta}\left(\eta_{\mu\nu} - \frac{1}{4\sum (p^6_{(2)})^2} \times \sum p_{(2)\mu} p_{(2)\nu} \frac{\exp\{\frac{i}{\hbar}[dS_2 + k_\lambda(x^\lambda_2 - x^\lambda_1)]\}}{\epsilon - k_\sigma k^\sigma}\right). \qquad (39)$$

Recall that the procedure of averaging over reference elements forming a classical macro-instrument, corresponds to using a set of frames of reference, i.e., in (38) and (39) one has to perform an integration in d^4k. Besides, one should take into account that $p^5_{(2)}$ corresponds to a charge of the object (2). As a result, (39) becomes the well-known relation for the electromagnetic vector potential at the charge (1) location point due to other ambient charges in the Fokker-Feynman direct particle interaction theory:

$$A_\mu(1) = \sum_{(2)} \int j_{(2)\mu} \delta(s^2(1,2)) ds_2. \qquad (40)$$

In the present case, instead of the δ function, we obtain the singular function

$$D^c_A(x_2 - x_1) = -\frac{1}{(2\pi)^4} \int \frac{\exp\{\frac{i}{\hbar} k_\lambda (x^\lambda_2 - x^\lambda_1)\}}{k_\sigma k^\sigma - \epsilon} d^4k, \qquad (41)$$

connected with the δ function in a known way. The small quantity ϵ determines herewith the integration contour.

7. The gravitational interaction

Let us consider separately the summands describing the gravitational interaction. To do that we neglect the electromagnetic interaction and write down the 5-dimensional metric with x^4 playing the role of the 5th coordinate:

$$dI^2_{(4)} = \tilde{G}_{AB} dx^A dx^B \neq 0. \qquad (42)$$

Let us perform a conformal transformation with the conformal factor found earlier from the form of the $\tilde{G}_{55}$ component:

$$\mathcal{F}_g = \frac{1}{2}(\epsilon - k_\sigma k^\sigma) \sum_{(2)} (p^6_{(2)})^2. \qquad (43)$$

We further perform a (1+4) splitting of the 5-dimensional metric by the prescriptions of the monad method in the chronometric type gauge. As there are off-diagonal metric components $G_{4\mu}$, the physically interpretable 4-dimensional metric takes the form

$$\breve{g}_{\mu\nu} = \sum \frac{\exp[\frac{2i}{\hbar} k_\lambda x^\lambda_1\}]}{\epsilon - k_\sigma k^\sigma} \times \left(\eta_{\mu\nu} + \frac{1}{8\sum (p^6_{(2)})^2} \sum_{(2)} [(p^5_{(2)})^2 + (p^6_{(2)})^2] \times p_{(2)\mu} p_{(2)\nu} \frac{\exp[\frac{i}{\hbar} k_\alpha (x^\alpha_2 - x^\alpha_1)}{\epsilon - k_\beta k^\beta}\right). \qquad (44)$$

Here out of the parantheses is the expression containing information on the position of a selected object (particle) relative to the origin of the effective frame of reference, while inside the parantheses there are the Minkowski metric components and the contributions to the metric from all other particles of the world. It is natural to assume that the metric of the Einstein general relativity coincides with the expression in the parantheses. Recalling that the summation sign includes averaging over all BISCR reference elements which form a macro-instrument, we arrive at the 4-dimensional metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{C}{8\sum (p^6_{(2)})^2} \sum_{(2)} [(p^5_{(2)})^2 + (p^6_{(2)})^2] \times p_{(2)\mu} p_{(2)\nu} \int \frac{\exp[\frac{i}{\hbar}[k_\lambda (x^\lambda_2 - x^\lambda_1)]}{\epsilon - k_\sigma k^\sigma} d^4k, \qquad (45)$$

where C is some dimensional coefficient.

Note that this expression corresponds to a representation often used in general relativity, namely, that of a Riemannian metric as the Minkowski metric plus small corrections:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad (46)$$

where the quantities $h_{\mu\nu}$ are quadratic in the 4-dimensional velocities of the sources, e.g., for dust-like matter they are proportional to the components of the energy-momentum tensor.

It should be also noted that in direct gravitational interaction theory, taken in its main approximation in the gravitational constant, one also obtains an effective Riemannian metric of the form (44), where $h_{\mu\nu}$ contains componentwise integration of the sources' energy-momentum tensor.

It is not difficult to unify the formulas of the last two sections, that results in a unified 6-dimensional theory of gravi-electromagnetic interactions. To do that, it is necessary to use a (1+1+4) splitting in the frames of the diadic method in a double chronometric gauge [15]. Then the curved 4-dimensional metric components will contain contributions from the electromagnetic terms.

8. Conclusion

1. The proposed theory may be treated as a generalization and development of the action-at-a-distance conception in physics, in particular, the Fokker-Feynman direct particle interaction theory. In the present approach, however, the distant action idea is extended to include the space-time concepts, while the previous studies of this kind assumed the space-time to be specified *a priori*

2. The present approach sheds some light on the essence of additional dimensions in Kaluza-Klein type multidimensional theories. They turn out to be stipulated by additional parameters of binary complex relation systems of ranks greater than (3,3).

3. The theory presented has provided a further development to the idea contained in Rumer's 5-optics, where it was suggested to use the classical action as the fifth coordinate. (Note that the squared interval of a 4-dimensional Riemannian space-time can be viewed as a 5-optics condition.) It then turns out, however, that Rumer's 5-optics is unsuitable for describing the electromagnetic interaction: one more dimension is necessary for that purpose. On the other hand, the x^4 coordinate was not so far considered in conventional Kaluza-Klein theory and the argument started from the curved metric components $G_{\mu\nu}$, actually already obtained with its aid.

4. The theory suggested solves the enigma which hampered for long the perception of the 5th dimension, namely, the essence of the G_{55} component. Different hypotheses were discussed in this respect, such as an additional scalar field of geometric origin, variations of physical constants, etc. In the present approach G_{55} plays a key role in the whole physics: it determines the denominators of the singular functions contained in the definition of intermediate bosons, the interaction carriers.

5. The present approach has clearly revealed the ideas commonly connected with Mach's principle in its wide meaning, i.e., the influence of the world's global factors upon local laws and phenomena. This is indicated by many aspects of the present approach: the additional parameters interpretation, the definition of the Riemannian metric, the meaning of the factor ϵ and others. It was not by chance that Einstein raised Mach's considerations to the rank of a principle in determining the curved space-time metric [17].

6. It is important to stress that the metric (gravitational interaction) definition contains a significant contribution from the 6th component of the multidimensional momentum p^6, determining the interaction of leptons with Z bosons in the algebraic model of electroweak interactions [4].

7. To conclude, the present paper has considered a transition from binary geometrophysics to multidimensional geometric models of physical interactions in the frames of the rank (4,4) BISCR. In higher rank theories, able to describe baryons, the treatment should remain essentially the same, just the formulas will be more cumbersome in appearance.

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