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PRINCIPLE OF OBSERVABILITY, CONSTRAINTS AND QUANTUM TO CLASSICAL TRANSITION

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The Leibniz-Mach approach (the description of Nature with only relative concepts) manifests itself in quantum theory as constraint conditions or, in other words, as a reduction of the physical phase space. In this paper the possibility of a classical-like behaviour is investigated for some simple toy models with constraints in terms of the Vilkovisky-DeWitt effective action in the one-loop order. It is shown that background fields, i.e., "empty" (with zero external currents) solutions of the classical dynamic equations are quantum-unstable. This rather "Machian" effect is explored for the Lense-Thirring model when only "rotational" modes of the gravitational field are quantized. The main result is: quantum corrections to the "diagonal" Einstein equations are divergent when the mass of a Lense-Thirring shell is zero and are suppressed in the case of "perfect dragging" of the inertial frame inside the shell.

1. Principle of Observability and quantum constraints

The "Principle of Observability" (PO) in its original version was enunciated by Leibniz as a principle of "Identity of Indiscernibles": "Two things which are completely indiscernible with respect to all their qualities are in fact the same thing and are therefore only one" (quoted from [1]). Berkley, Mach, Bridgman advocated this, after all, purely pragmatic principle, which is as a matter of fact just a maximally generalized relativity requirement: a "thing" becomes a reality only through its relations with other things. "In practical measurements we always do the same: we compare physical objects" (E. Mach, [2]). "Any admitted physical notion should be directly or indirectly connected — by a finite number of intermediate steps — with observable facts". This formulation of Mach's PO was given by Hönl [3] (Cf. Wheeler's "It from Bit" and the "Participatory Universe" approach [4]). Thus our language must be self-consistent, a "word" taken out from the context disappears, becomes a noise. From the point of view of PO, any exact symmetry is questionable. Imagine that the electromagnetic interaction is switched off — this will make isotopic symmetry exact and there will be no way to distinguish a proton and a neutron and we would never consider them as two different particles. According to PO, two quantum states, connected by some exact symmetry transformation, must be a single state rather than two; in other words, PO requires that only singlets, S-states which obey the constraint equations

$$\hat{Q}|\psi\rangle = 0 \tag{1}$$

may be realized in Nature. Conventional classical or quantum theory does not incorporate this requirement; it is indeed very strong. Let us consider the simplest example of a free nonrelativistic particle, whose action

$$S = \int \tfrac{1}{2} m\dot{x}^2\, dt \tag{2}$$

possesses the symmetry $x \to x + \text{const}$. Due to this symmetry, all points are indistinguishable and according to PO we must impose the constraint

$$\hat{p}_x|\psi\rangle = -i\frac{\partial}{\partial x}|\psi\rangle = 0, \tag{3}$$

which is of course much stronger than the momentum conservation $\dot{p}_x = 0$. The validity of (3) means that instead of (2) this "particle" is described by the gauge theory

$$S = \int \tfrac{1}{2} m(\dot{x} - y)^2\, dt \tag{4}$$

($y(t)$ is a Lagrange multiplier) with zero number of physical degrees of freedom (see e.g. [5]). No classical limit is possible here, since no wave packet may be formed with the solutions $\psi = \text{const}$ to the constraint (3).

In elementary particle physics the requirement that in the case of exact symmetry only the zero-charge subspace of the asymptotic Hilbert space is admissible, is also rather strong. In practice we always observe either breakdown, or confinement of symmetries. The

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Abelian $U(1)$ symmetry of electrodynamics is perhaps the only exception from this rule. The coloured states ($Q \neq 0$) of the exact SU(3) symmetry of quantum chromodynamics cannot “fly out”, whereas many other symmetries are violated. More than that: zero mass Goldstone bosons, which manifest vacuum degeneracy in spontaneous symmetry breaking models, are never observed as well; they happen to be massive, but their masses are now introduced to the theory extrinsically. As we shall see below (Sec. 4), the “quantum repulsion” from degenerate vacua backgrounds results in a dynamical creation of a Goldstone boson mass. Earlier one of the authors (B.A., [6]) proposed the “breakdown or confinement” dilemma as a formulation of the generalized Mach’s principle. In this paper we try to introduce Mach’s principle to quantum cosmology through the mechanism of quantum renormalization of the classical vacuum, which manifests itself dynamically as a “quantum repulsion” from a degenerate vacuum state.

Let us discuss now the space-time Poincaré symmetry. We know that it is extrinsically broken on the cosmological level: there is a special frame comoving with the background radiation, such that its temperature is isotropic. At the same time, this frame is an inertial one; this coincidence is always referred to by the advocates of Mach’s principle (“No space-time may exist without matter of the Universe”) as an experimental proof of its validity. In modern words “Mach’s paradox” is just a dissatisfaction of the situation of a degenerate vacuum: in empty space there is nothing observable that determines the choice of the inertial frames. This “paradoxical” freedom in the choice of the asymptotic gravitational field (“absolute space”) is absent in the closed Universe invented by Einstein as an answer to Mach’s challenge.

The Wheeler-DeWitt constraint of quantum cosmology

$$\hat{H}|\psi\rangle = 0 \tag{5}$$

reflects the relativity with respect to the time coordinate. The “shift” constraints

$$\hat{M}^i|\psi\rangle = 0 \tag{6}$$

tell the same about rotations. Barbour [7] advocates the viewpoint that the constraints (5) and (6) select Machian space-times (see also [8]). Here we will try to show the same within the frames of quantum field theory: “empty” spaces do not exist in the classical limit of systems with quantum constraints.

2. The classical limit and quantum constraints

A quantum system may be considered to be approximately classical if quantum dispersion is small compared with the averages of observables. In particular, the classical space-time of our Universe, with its almost definite space-like or time-like distances between the world points, exists because quantum fluctuations of the metric are much smaller than $g_{\mu\nu}$ itself. In this paper we will use the language of quantum effective action (EA) to approach the problem of quantum to classical transition for systems with constraints. The one-loop (lowest in $\hbar$) terms in the EA, as well as tadpole diagrams, i.e., the lowest quantum corrections to the classical dynamic equations, will be studied. The divergency of these one-loop corrections indicates the absence of a classical limit.

It is well-known that a restriction of the Hilbert space to some subspace results in nontrivial kinematics and dynamics of physical observables and also results in problems with the classical limit (see, e.g., the review [9]). If we postulate, for instance, that only even states ($\psi(x) = \psi(-x)$) of the most popular one-dimensional oscillator are permitted, then $|x|$ instead of x becomes the relevant physical observable and, contrary to the conventional oscillator model, there is no classical limit near $x = 0$. The same is true for the dynamics of the radial coordinate of a free particle constrained to the S state of its Hilbert space [10]; the Van der Monde determinant and the phenomenon of quantum “level repulsion” repeat the story for unitary matrices averaged over angles [11]. In quantum cosmology, the zero energy WDW constraint (5) resulted in a flood of papers with more or less unique conclusion: the construction of time and the classical space-time is impossible from the pure wave function of the Universe; the system must be open, some environment is necessary to provide decoherence and hence the wave packet reduction to “Cat Alive”, i.e., to an approximately classical space-time, the room for our living and putting questions.

For a quantum-field theorist, the introduction of an environment just means that the gauge freedom is broken “by hand”: the best way to solve the problem is to erase the question. One can use as well the unitary gauge ($x^0 = t$ or $a = t$; a is the scale factor of the Universe, see ,e.g., the review [12]), which just means that one of the quantum coordinates is declared to be nonquantized and is nominated to be a classical clock, i.e. to be a classical reference system. Or one can introduce Mensky channels [13], i.e., perform the substitution (symbolically)

$$\delta(H) = \int \mathrm{e}^{iNH}\, dN \;\Rightarrow\; \int \mathrm{e}^{-\gamma N^2} \mathrm{e}^{iNH}\, dN, \tag{7}$$

etc. All these approaches just mean that the problem of the classical limit in quantum cosmology is erased by the will of the author of the paper. Here we consider the gauge freedom (and the global “residual gauge freedom”, see below) seriously and will not eliminate it artificially. It will be shown that in terms of the EA the presence of a constraint results in “quantum repulsion” from the mass shell; that means that the EA possesses a specific infrared non-analyticity or divergency at the points of the functional space which are solutions of the classical dynamical equations. The necessity of a nonzero RHS of these equations (i.e., the necessity of external currents) is in a sense a counterpart of the introduction of the above mentioned environment. But in our approach it is not done arbitrarily but results from an unambiguously calculated form of the EA.

3. Feynman functional integral and constraints. The role of "Residual Gauge Freedom"

Let us look at the well known chain of expressions which are valid in the minisuperspace models of quantum gravity and in relativistic particle theory:

$$\int \mathrm{e}^{i\int (p_i\dot{q}^i - H)\,dt} \prod_t \delta(H_t)\, DpDq$$

$$= \int \mathrm{e}^{i\int (p_i\dot{q}^i - H)\,dt + i\int NH\,dt}\, DpDqDN \tag{8}$$

$$= \int \mathrm{e}^{i\int (T/N - NU)\,dt} M\, DqDN \tag{9}$$

$$\simeq \int \mathrm{e}^{2i\int \sqrt{-TU}\,dt} M\, Dq. \tag{10}$$

Here T is the kinetic energy, U is the potential, $H = T + U$, M is a measure. The last line resulted from substituting the extremal value of N

$$N_{\mathrm{extr}} = \sqrt{\frac{-T}{U}} \tag{11}$$

to (9). The Jacobi-type action in (10) was first written down in general relativity by Baierlein, Sharp and Wheeler [14]; see the discussion in [7]. Of course, Eqs. (8)–(10) make sense only if a gauge fixing term is introduced before the functional integration. However if we overfix the gauge freedom (see e.g. (7)), then we shall loose the constraint. Teitelboim [15] and Halliwell [16] introduced the gauge fixing term $\delta(\dot{N})$ and showed that the WDW equation (5) is preserved due to ordinary integration over residual gauge freedom $N = \mathrm{const}$; $N = \mathrm{const}$ is just a zero of the Faddeev-Popov ghost operator for the gauge condition $\dot{N} = 0$. (Note: if we take this gauge, then in (10) we have another form of the Jacobi-type action:

$$\int \sqrt{-TU}\,dt \;\Rightarrow\; \sqrt{-\int T\,dt \cdot \int U\,dt'}, \tag{12}$$

thus the arbitrariness discussed in (7) results from the choice of gauge fixing).

Here we shall not fasten ourselves, as is done in (10) and (12), to the extremal values of the Lagrange multipliers; we will consider them and the dynamical variables q^i on the equal grounds and use in (9) the relativistic, so-called "natural" Landau-DeWitt gauge. The use of this gauge is an important ingredient in the definition of Vilkovisky-DeWitt unique effective action (VDWEA), see Appendix. In the subsequent sections we calculate the one-loop terms of VDWEA in some simple toy models with constraints. The nontrivial point to compare with standard procedure is that the Gaussian functional integrations, which determine Green functions and loops, are redefined in a way to take into account the residual gauge freedom.

This supplementary integration is equivalent to the requirement that the wave function be invariant under "large" diffeomorphisms, the zeros of the Faddeev-Popov determinant. This requirement is indeed physically nontrivial. For example, in QED an integration over the "large" gauge transformation $\xi = Ct$ (i.e., $A_0 \to A_0 + C$) gives the condition of zero charge (1), which is of course stronger than the Gauss law constraint. In quantum gravity an integration over angular velocities Ω (i.e., over the "large" gauge transformation $\varphi \to \varphi + \Omega t$) will result in the zero-momentum constraint (6), etc. (see also [17]).

The residual gauge contribution to the functional integration is responsible for the specific IR divergencies regularized by the external currents,

$$j_{\mathrm{ext}\,A} \equiv \frac{\delta S}{\delta \varphi^A} \tag{13}$$

hence we have a "quantum repulsion" from the free, "empty" solutions to the classical equations. Let us demonstrate that on simple examples.

4. Higgs model with an Abelian gauge field

The physics of the model is determined by the action

$$S = \int \Big\{ -\tfrac{1}{4} F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}\big|\varphi_{,\mu} - ieA_\mu\varphi\big|^2 - V\big(|\varphi|\big) \Big\}\, d^4x, \tag{14}$$

and the functional space interval (A1)

$$dl^2 = \int \Big\{ -dA_\mu dA^\mu + d\varphi d\varphi^* \Big\}\, d^4x - \int \Big\{ -dA_0^2 + \sum_{k=1}^{3} dA_k^2 + d\rho^2 + \rho^2 d\beta^2 \Big\}\, d^4x \tag{15}$$

where $\varphi = \rho \mathrm{e}^{i\beta}$; the signature is $(+\,-\,-\,-)$; the potential

$$V(\rho) = \frac{m^2}{8\rho_0^2}\left(\rho^2 - \rho_0^2\right)^2 \tag{16}$$

has the standard symmetry-breaking form, m is the mass of the ρ field at the minimum $\rho = \rho_0$ of V, $m \ll \rho_0$. We shall consider $\rho = \mathrm{const}$ as a background and quantize the gauge (A_μ) and Goldstone (β) fields by the VDWEA method. (Eqs. (3) and (4) give the simplest quantum-mechanical analogy for this model if we put $A_k = 0$, $\rho = 1$ and identify $x(t) \leftrightarrow \beta(t)$, $y(t) \leftrightarrow A_0(t)$.)

The functional Christoffel symbols for the metrics from the last expression in (15) are nonzero; for the $\{A_\mu,\ \rho, \beta\}$ field parametrization the term

$$\left\{ \begin{array}{c} \rho \\ \beta\beta \end{array} \right\} S_{,\rho} = \rho \frac{dV}{d\rho} \equiv \rho^2 \mu^2 \tag{17}$$

in $S_{;AB}$ (see (A3)) gives a nonzero (outside the mass shell) mass μ of the Goldstone boson $\beta(x)$.

According to the general rules (see the Appendix) and the gauge transformations

$$\delta A_\mu = \xi_{,\mu}; \quad \delta\beta = e\xi, \tag{18}$$

we obtain the natural gauge condition

$$A^\mu_{,\mu} + e\rho^2\beta = 0, \tag{19}$$

the residual gauge equation

$$\Box\xi + \mathrm{e}^2\rho^2\xi \equiv \hat{Q}\xi = 0 \tag{20}$$

and the following expression for the one-loop term of the VDWEA:

$$\Gamma^{(1)}(\rho) = \lim_{\alpha\to\infty} \Omega^{(4)} \int d^4p \left\{ -\tfrac{1}{2} \ln\det \hat{E}(p) + \ln\left[\alpha^{1/2} Q(p)\right] \right\}. \tag{21}$$

Here $\Omega^{(4)}$ is the four-volume. $\Gamma^{(1)}$ was calculated according to (A8–10), where $\eta^B = \{A_\mu; \beta\}$,

$$S^{(2)} = \tfrac{1}{2}\eta^B E_{BC}\eta^C = \int \Big\{ -\tfrac{1}{4}F_{\gamma\nu}F^{\gamma\nu} + \tfrac{1}{2}\rho^2(\beta_{,\gamma} - eA_\gamma)^2 - \tfrac{1}{2}\mu^2\rho^2\beta^2 + \tfrac{\alpha}{2}(A^\gamma_{,\gamma} + e\rho^2\beta)^2 \Big\} d^4x, \tag{22}$$

$$\hat{Q}^{-1} = (\Box + \mathrm{e}^2\rho^2)^{-1} \Rightarrow Q^{-1}(p) = (p^2 - \mathrm{e}^2\rho^2)^{-1}, \tag{23}$$

$\hat{Q}^{-1}$ is the ghost propagator. In the limit $\alpha = \infty$ the Green function $\hat{D}(p) = [\hat{E}(p)]^{-1}$ has physical "transverse" poles at

$$p^2 = \mathrm{e}^2\rho^2 \tag{24}$$

and "longitudinal" poles at

$$p^2 = m^2_{1,2} = \tfrac{1}{2}\mu^2 + \mathrm{e}^2\rho^2 \pm \tfrac{1}{2}\sqrt{\mu^4 + 4\mathrm{e}^2\rho^2\mu^2}, \tag{25}$$

which coincide at $\mu = 0$.

From (21) we have

$$\frac{d\Gamma^{(1)}}{d\mu^2} \sim \int \frac{d^4p}{(p^2 - m_1^2)(p^2 - m_2^2) - i\varepsilon} = \frac{1}{m_1^2 - m_2^2}\Big(D^c_{m_1}(x) - \overline{D}^c_{m_2}(x) \Big)_{x=0}. \tag{26}$$

This expression is divergent at $\mu \to 0$ like μ^{-1}, and in this sense it drastically differs from the conventional one:

$$\int \frac{d^4p}{(p^2 - m_1^2 - i\varepsilon)(p^2 - m_2^2 - i\varepsilon)} = \frac{1}{m_1^2 - m_2^2}\left(D^c_{m_1} - D^c_{m_2} \right)_{x=0}. \tag{27}$$

That is the point: due to the presence of the anti-casual Green function in (26), the residual gauge freedom is taken into account, — hence we have the divergence of (26) at zero Goldstone boson mass μ. We are not interested in a struggle with ultraviolet divergencies in (26). The phenomenon of quantum repulsion from the mass shell is of infrared nature. Thus we calculate only the pole contribution in (26), which finally gives the following one-loop correction to the effective potential:

$$V^{(1)} \equiv \frac{\Gamma^{(1)}}{\Omega^{(4)}} = -\int_0^{\mu^2} \frac{\tfrac{3}{4}\mu^4 + 3\mathrm{e}^2\rho^2\mu^2 + 2\mathrm{e}^4\rho^4}{4\sqrt{\mu^4 + 4\mathrm{e}^2\rho^2\mu^2}}\, d\mu^2. \tag{28}$$

Eq. (28) is valid for any form of $V(\rho)$, i.e., $\mu(\rho)$ in (17). For small μ

$$\mu^2 \ll \mathrm{e}^2\rho^2 \tag{29}$$

(i.e., according to (17), for $dV/d\rho \ll \mathrm{e}^2\rho^3$). We obtain from (28) the effective potential:

$$V_{\text{eff}}(\rho) = V(\rho) + V^{(1)}(\rho) = V(\rho) - \tfrac{1}{2}\mathrm{e}^3\rho^3\sqrt{\frac{1}{\rho}\frac{dV}{d\rho}}. \tag{30}$$

For the potential (16) near the classical vacuum $\rho = \rho_0$, Eq.(30) gives:

$$V_{\text{eff}}(\rho) \approx \tfrac{1}{2}m^2(\rho - \rho_0)^2 - \tfrac{1}{2}\mathrm{e}^3\rho_0^3 m\sqrt{\frac{\rho}{\rho_0} - 1}. \tag{31}$$

The non-analytic behaviour of V_{eff} at $\rho = \rho_0$ is analogous to the Coleman-Weinberg effect of spontaneous symmetry breakdown by the radiation corrections [18]. Now we have a spontaneous origin of the Goldstone boson mass. Indeed, the potential (31) has a minimum at

$$\overline{\rho} = \rho_0 + \mathrm{e}^2\rho_0\left(\frac{\rho_0}{4m}\right)^{2/3}, \tag{32}$$

and at this extremal point the Goldstone boson mass is

$$\mu \equiv \sqrt{\frac{1}{\rho}\frac{dV}{d\rho}} = em\left(\frac{\rho_0}{4m}\right)^{1/3}. \tag{33}$$

The physics behind these results is rather simple. Any model of spontaneous symmetry breakdown puts the same question: "Who selects one of many degenerate "broken" vacua, with a definite phase of the order parameter?" Bogolubov's quasiaverages are the most popular answer. But that means that the system is no more closed (cf. the "environment" mentioned in Sec. 2).

Our postulate is: the system is at any moment in the S state with respect to the phase β. That is why it is repelled from the $\rho = \rho_0$ ($\mu = 0$) state. Stabilization is reached by a dynamically induced displacement from the classical vacuum $\rho = \rho_0$, which also provides a mass to the Goldstone boson.

5. One-loop VDWEA term for a relativistic particle

A relativistic particle is the simplest model for quantum cosmology [12]. The action and the functional metric (A1) are

$$S = \int \left(\mathrm{e}^{-\nu}T - \mathrm{e}^{\nu}U\right)\, dt, \tag{34}$$

$$dl^2 = \int \mathrm{e}^{\nu}\left\{d\nu^2 + \gamma dq_i dq^i\right\}\, dt, \tag{35}$$

where $N \equiv \mathrm{e}^{\nu}$ is the lapse function; $T = \frac{1}{2}\dot{q}_i\dot{q}^i$ ($i = 1,\ 2,\ \ldots,\ n$); $U = \mathrm{const} \sim m$; $\gamma = \mathrm{const}$, $[\gamma] = \mathrm{cm}^{-1}$. This is the simplest general relativity where only $g_{00}(t) \equiv N^2$ component of space-time metrics is left.

The time reparametrization $t \to t + \xi(t)$ generates the gauge transformation

$$\delta\nu = \dot{\nu}\xi + \dot{\xi}, \quad \delta q^i = \dot{q}^i\xi. \tag{36}$$

We consider the background

$$\overline{\nu} = 0, \quad \dot{\overline{q}}^i = v^i = \mathrm{const}. \tag{37}$$

Instead of passing from (34) (or (9)) to (10) or (12), we shall calculate a one-loop term considering small field variations η^ν, η^i on equal grounds in the relativistic natural gauge (A5):

$$\dot{\eta}^\nu + \gamma v^i\eta_i = 0. \tag{38}$$

The residual gauge equation is

$$\ddot{\xi} + 2\gamma\overline{T}\xi \equiv \hat{Q}\xi = 0. \tag{39}$$

The external currents at the background are

$$S_{,\nu} = -\left(\overline{T} + U\right) = -\left(\tfrac{1}{2}v_i v^i + U\right) \equiv -\overline{H}, \tag{40}$$

$$S_{,q^k} = -\ddot{\overline{q}}_k - U_{,k} = 0. \tag{41}$$

The operator $\hat{E}$ in (A9) in the momentum representation is ($\eta^A = \{\eta^\nu, \eta^i\}$)

$$\hat{E}(p) = \begin{array}{c} \\ \eta^\nu \\ \eta_k \end{array}\!\!\begin{array}{c} \qquad\eta^\nu \qquad\qquad\qquad\qquad \eta^j \\ \left(\begin{array}{c|c} \frac{3}{2}\overline{T} - \frac{1}{2}U + \alpha p^2 & -ip(1+\alpha\gamma)v^j \\ \hline ip(1+\alpha\gamma)v_k & \left[p^2 - \frac{\gamma}{2}(\overline{T}+U)\right]\delta^j_k + \alpha\gamma^2 v^j v_k \end{array}\right) \end{array} \tag{42}$$

and (as $\alpha \to \infty$) possesses the eigenvalues

$$\lambda_1 = \alpha\left(p^2 + 2\gamma\overline{T}\right) \tag{43a}$$

$$\lambda_2 = \frac{\left(p^2 - 2\gamma\overline{T}\right)^2 - \frac{\gamma}{2}\overline{H}\left(p^2 + 2\gamma\overline{T}\right)}{p^2 + 2\gamma\overline{T}} \tag{43b}$$

$$\lambda_3^{\perp} = p^2 - \tfrac{\gamma}{2}\overline{H}, \quad (n-1)\text{−degenerate}. \tag{43c}$$

According to the standard procedure,

$$\Gamma^{(1)} = \Theta\int\left\{\tfrac{1}{2}\ln\alpha + \ln\!\left(p^2 - 2\gamma\overline{T}\right) - \tfrac{1}{2}\ln\prod_\kappa \lambda_\kappa\right\} dp, \tag{44}$$

$$\frac{d\Gamma^{(1)}}{d\overline{H}} = \frac{1}{2}\Theta\int\left\{\frac{\frac{\gamma}{2}\left(p^2 + 2\gamma\overline{T}\right)}{(p^2 - m_1^2)(p^2 - m_2^2) - i\varepsilon} + \frac{\frac{\gamma}{2}(n-1)}{p^2 - \frac{\gamma}{2}\overline{H} - i\varepsilon}\right\} dp, \tag{45}$$

$$m_{1,2}^2 = 2\gamma\overline{T} + \tfrac{\gamma}{4}\overline{H} \pm \tfrac{\gamma}{2}\sqrt{\overline{H}\left(\tfrac{1}{4}\overline{H} + 8\overline{T}\right)}. \tag{46}$$

See the definition of $\overline{H}$ and $\overline{T}$ in (40). $\overline{H}$ is a measure of departure from the classical constraint $\overline{H} = 0$. For $\overline{H} \ll \overline{T}$, calculating $\Gamma^{(1)}$ from (44) and (45), we obtain the effective action as a function of the background fields $v^i, \overline{\nu}$ (we restore here $\mathrm{e}^{\overline{\nu}} = \mathrm{const}$, the lapse function which is necessary to provide the invariance of (47) with respect to the reparametrizations $t \to t + ct$ which do not violate the constancy of $v^i, \overline{\nu}$):

$$\Gamma_{\text{eff}} = \int\left\{\mathrm{e}^{-2\overline{\nu}}\,\overline{T} - U + c\sqrt{\mathrm{e}^{-2\overline{\nu}}\,\overline{T} + U}\right\}\mathrm{e}^{\overline{\nu}}\, dt, \tag{47}$$

$$c = \frac{\sqrt{2} + n - 1}{\sqrt{8}}\sqrt{\gamma}. \tag{48}$$

If we change the gauge condition from (38) to the Teitelboim-Halliwell one $\dot{\eta}^\nu = 0$, then the only change in Γ_{eff} will be in the value of c in (48): $\sqrt{2}$ will be replaced by 1.

By (47), we can write the new "effective" constraint

$$\frac{\partial\Gamma_{\text{eff}}}{\partial\overline{\nu}} = 0 \;\Rightarrow\; \overline{H} - \frac{cU}{\sqrt{\overline{H}}} = 0, \tag{49}$$

which yields the renormalized value of the total energy

$$\overline{H}_0 = (cU)^{2/3}. \tag{50}$$

Again, just as in the Abelian field Higgs model of Sec. 4, we obtain a "repulsion" of Γ_{eff} (47) from the classical dynamical equation $\mathrm{e}^{-2\overline{\nu}}T + U = 0$. The renormalization of classical equations is typical in quantum theory; here it is provided by the quantization of the "residual" gauge modes which are solutions to (39).

6. "Perfect dragging" from quantization of gravity

A thin massive (M) slowly rotating spherical (of radius r_0) shell drags the inertial frame inside it. The heavier is the shell, the stronger is the dragging. When the gravitational radius $r_g = 2kM$ reaches r_0, the dragging becomes "perfect", i.e., the inertial frame inside the shell is fastened to it [19], [20]. In this case the non-Machian arbitrariness of the inertial frame inside the shell is absent, the quantum state of the gravitational field is gauge invariant under the gauge transformations

$$\varphi \to \varphi + \Omega t. \tag{51}$$

We have studied quantum corrections to the background standard spherically symmetric solution of the Einstein equations in $N = D + 2$ dimensions in the presence of a massive shell when only the "rotational" components of the space-time metric are quantized. The best tool is the Kaluza-Klein reduction $M^N = M^2 \times S^D$, which gives the 2-dimensional gravity $g_{\alpha\beta}$ with the dilaton ψ and the Abelian vector field A_μ ($\mu = 0, 1$) (see e.g. [21]):

$$S_{\rm red} = \frac{1}{2k} \int \left\{ R^{(2)} + D(D-1) \frac{\psi^{,\mu}\psi_{,\mu}}{\psi^2} - \frac{D(D-1)}{\psi^2} - \frac{1}{4}\psi^2 F_{\mu\nu}F^{\mu\nu} \right\} \psi^D \sqrt{-g^{(2)}}\, d^2x, \tag{52}$$

where $x^\mu = \{t, r\}$; $F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$; $A_\mu \equiv A_\mu^{(D)}$: of all the Kaluza-Klein vector fields, only one, corresponding to rotations along the azimuthal angle $\varphi \equiv x^{(D)}$, is left to be nonzero. This field is quantized, whereas $g_{\alpha\beta}$ are ψ are treated as a background, just as ρ in the Abelian Higgs model of Sec. 4. To perform the VDW calculations, a reduction of the VDW functional metric (A2) must be also carried out. Its A_μ part

$$dl^{2(A)} = \int 2\psi^{D+2} \sqrt{-g^{(2)}} g^{\mu\nu} dA_\mu dA_\nu d^2x \tag{53}$$

depends on the background fields $g_{\alpha\beta}$ and ψ just as $dl^{2(\beta)}$ depends on ρ in (15). The Landau-DeWitt gauge (A–5)

$$\left(\psi^{D+2} A^\mu\right)_{;\mu} = 0 \tag{54}$$

gives the Faddeev-Popov ghost operator (cf. (20), (39))

$$\hat{Q} = \nabla_\mu \left(\psi^{D+2}\, \partial^\mu\right). \tag{55}$$

Quite analogously to the models of Secs. 4 and 5, the longitudinal part of the Green function is proportional to

$$\hat{D} \sim \left[\hat{Q}^2 - M\delta(r - r_0)\hat{\triangle} - i\varepsilon\right]^{-1}, \tag{56}$$

$\hat{\triangle}$ being some differential operator of second order in d/dr. It is again easily seen that the absence of external matter ($M = 0$) yields a divergence of $\hat{D}$ caused by the double ghost pole in (56).

We did not study these formulae to the end but considered the gauge condition $A_1 = 0$ which is much simpler than (54). In this case a quantum averaging over the residual gauge mode

$$A_0 = \Omega, \tag{57}$$

which corresponds to a uniform rotation (51), in the presence of a massive shell gives the following result for the tadpole corrections to the Einstein equations inside the shell:

$$\left\langle \hat{T}_{\alpha\beta} \right\rangle_{A_\mu} \sim \frac{1}{M}\left(1 - \frac{2kM}{r_0}\right). \tag{58}$$

Thus for the empty space ($M = 0$) these quantum fluctuations are infinite, while in the perfect dragging situation ($2kM = r_0$) they vanish.

7. Summary and questions

1. Many years ago one of the authors' 3-year-old son pestered him with the question: "Why is there NEAR and FAR?" This question may be also generalized to: "Why is there LONG AGO and RECENTLY?" The answer "Because there is more or less definite space-time metric" meets the following questions: "What determines the metric?", and "Why are its quantum fluctuations much smaller than 1?" Mach's principle, as Einstein put it, says "... all inertia, that is, all the g_{ik} field, is determined by matter in the universe and not by the boundary conditions at infinity" [22]. A closed universe does not possess the non-Machian arbitrariness of metric at infinity. The situation of "perfect dragging" (see Sec. 6) is another popular example of a Machian situation inside a shell. From this point of view Eq. (58) is even more informative than just "quantum repulsion" from the free ($M = 0$) solutions to the Einstein equations. It shows that for $r_g = 2kM = r_0$ the gravitational field inside the shell becomes gauge-singlet and no room is left for the non-Machian arbitrariness of the inertial frame. The background space-time which possesses such a "confinement", is rather special. The integral formulation of the Einstein equations, proposed earlier by one of the authors and other people [23], is a "selection rule" approach to Mach's principle. We tried here to find a dynamic justification for this selection, considered so far by many physicists as a "philosophical" and unnecessary one. The quantum effective action, calculated in this paper with averaging over residual gauge modes, is nonlocal. Is it possible to deduce the integral formulation of the Einstein equations from this EA? This topic is discussed in more detail in Ref. [24].

2. In Ref. [25] the idea was expressed that a spontaneous breakdown of general covariance (SBGC), which gives a nonzero space-time metric $\langle \hat{g}_{ik} \rangle$ (cf. [26]), will automatically lead to Machian spaces and exclude non-Machian ones. But this is not true. By full analogy with the Higgs model of Sec. 4, where conventional spontaneous breakdown of the Abelian symmetry leads to a degenerate vacuum $\rho = \rho_0$ with some definite, although unpredictable, phase $\langle \hat{\beta} \rangle$, SBGC must lead to a non-Machian vacuum with an arbitrary choice of inertial frames (that is with some choice of the $\langle \hat{g}_{ik} \rangle$ asymptotics at infinity). A quantum averaging over

these asymptotics must give, as was shown above for the Higgs model, a nontrivial dynamics which forbids "empty" spaces $\langle \hat{g}_{ik} \rangle$. More work is necessary to go beyond the toy models and to show that for gravity itself.

3. The Principle of Observability demands that every physical concept and notion (every "object" and "word") of the theory must be identified by some observational procedure in the frames of dynamics of the same theory. Isn't it a challenge to give a rigorous formulation of this Self-consistency Law, a formulation which perhaps will make our theories more unique and predictive. In this work we postulated quantum constraints for conserved charges and investigated the nontrivial consequences. But why constraints? To describe them, we here used quantum functional averaging over global (nonzero at infinity) fields with zero or finite value of the classical action. But perhaps the logic might be turned upside down: instead of taking the ad hoc constraints (1), a universal procedure of "averaging" over all relations of a given "object" with other "objects" may be postulated in such a way that the final "dynamics" (an analogy of the effective action) forbid indiscernible objects and, in particular, forbid the exact symmetries?

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Appendix. The Vilkovisky-DeWitt effective action (VDWEA)

The conventional EA suffers from two ambiguities: (1) it depends on fields reparametrization; (2) in gauge theories its form (outside the mass shell) depends on gauge fixing. The VDWEA approach [27, 28] (see e.g. [29] and references therein) heals the first disease by considering the functional space φ^A (A is the condensed De Witt index, both discrete and continuous) as a Riemannian space with an interval

$$dl^2 = d\varphi^A G_{AB}(\varphi) d\varphi^B. \tag{A1}$$

In particular, for the gravitational field ($\varphi^A = g_{\mu\nu}$)

$$\begin{aligned} G_{AB} &= G^{\mu\nu\alpha\beta}\delta(x-x') \\ &= \tfrac{1}{2}\sqrt{-g}\big(g^{\mu\alpha}g^{\nu\beta} \\ &+ g^{\mu\beta}g^{\nu\alpha} + c g^{\mu\nu}g^{\alpha\beta}\big)\delta(x-x'). \end{aligned} \tag{A2}$$

Now, instead of ordinary functional derivatives, functionally covariant derivatives are used, e.g. (S is the classical action),

$$S_{,AB} \;\Rightarrow\; S_{;AB} = S_{,AB} - \left\{ {C \atop AB} \right\} S_{,C}\ , \tag{A3}$$

(the Christoffel symbols are built from the metric (A1)). Hence loops do not depend on the choice of coordinates in the functional space. In gauge theories the generators $K^A_\gamma(\varphi)$ of gauge transformations

$$\delta\varphi = K^A_\gamma \xi^\gamma \tag{A4}$$

(parametrized by the fields ξ^γ) must be Killing vectors of G_{AB}. At each point $\overline{\varphi}^A$ of the functional space we can take differentials η^A of the fields ($\varphi^A = \overline{\varphi}^A + \eta^A$) in the direction orthogonal to a gauge orbit:

$$K^A_\gamma G_{AB}\eta^B \equiv K_\gamma \eta = 0. \tag{A5}$$

This is the Landau-DeWitt, or the so-called "natural" gauge fixing. The substitution $\eta^B = K^B_\beta \xi^\beta$ in (A5) leads to the Faddeev-Popov operator

$$\hat{Q}_{\gamma\beta} = K^A_\gamma G_{AB} K^B_\beta, \tag{A6}$$

and to the equation for the "residual" gauge transformations $\xi^\beta_{(0)}$

$$\hat{Q}_{\gamma\beta}\xi^\beta_{(0)} = 0. \tag{A7}$$

In the most simplified formulation, the second VDWEA postulate which makes the EA unique, is: use the "natural" gauge (A5). To tell strictly, this is true only for one-loop terms; the special Vilkovisky connection [28] must be used to calculate the unique VDWEA in higher orders in $\hbar$. But in this paper we are interested only in the one-loop term $\Gamma^{(1)}$, so we use (A5) to calculate it. Thus,

$$e^{i\Gamma^{(1)}} = \lim_{\alpha\to\infty} \int e^{iS^{(2)}} \operatorname{Det}\hat{Q}\cdot M\left[D\eta^A\right], \tag{A8}$$

where $S^{(2)}$ is a sum of a second functionally covariant variation of S and a gauge fixing term:

$$\begin{aligned} S^{(2)} &\equiv \tfrac{1}{2}\eta^A E_{AB}\eta^B \\ &= \tfrac{1}{2}\eta^A S_{;AB}\eta^B + \tfrac{\alpha}{2}(K_\gamma\eta)c^{\gamma\beta}(K_\beta\eta), \end{aligned} \tag{A9}$$

$M = \big[\operatorname{Det}(\alpha c^{\gamma\beta})\big]^{1/2}$. Finally, we obtain the one-loop VDWEA

$$\begin{aligned} \Gamma_{\text{eff}}(\overline{\varphi}) &= S(\overline{\varphi}) + \Gamma^{(1)}(\overline{\varphi}) \\ &= S(\overline{\varphi}) - \lim_{\alpha\to\infty} \ln\{\tfrac{1}{2}\operatorname{Det}\left(E_{AC}G^{CB}\right) \\ &\qquad + \ln\operatorname{Det}\hat{Q} + \ln M\}. \end{aligned} \tag{A10}$$

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