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ON A POSSIBLE GEOMETRIC DESCRIPTION OF BROKEN FAMILY SYMMETRY

A.S. Sakharov† and E.K. Sarkisyan[1]‡

† Center for Cosmoparticle Physics “Cosmion”, Miusskaya pl., 4, 125047, Moscow, Russia

‡ School of Physics and Astronomy, Tel Aviv University, Ramat Aviv 69978, Tel Aviv, Israel

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A scheme of geometric interpretation ($M^{\times} S^1 \times S^1 \times S^1 \times S^1$) of the theory of broken gauge symmetry of families is constructed.

The ideas of Kaluza on physical interactions as manifestations of additional space-time dimension were generalized in [1]. This generalization allowed one to give a geometric interpretation of most conventional models of particle physics. The basic elements of the standard gauge description of physical interactions have been included in the geometrical models (the 6-dimensional model of electroweak interactions and the 7-dimensional one for QCD) in the framework of a classical geometric approach. However, modern particle theories contain elements and mechanisms unused in standard models. Thus there arises the necessity of applying a geometrical approach to more complicated particle theories. In this work we propose to apply the ideas of [1] to the theory of spontaneously broken gauge family symmetry, which is a basis for a quantitative phenomenological description of all the phenomena of particle physics and cosmology (see [2]).

The $SU(3)_H$ model of spontaneously broken local family symmetry (BFS) is considered to be the simplest version of realistic quantum flavour dynamics, giving a reasonable explanation of the mass hierarchy and the mixing pattern of quarks and leptons. The concept of local horizontal symmetry $SU(3)_H$ with left-handed quark and lepton components being $SU(3)_H$ triplets and right-handed ones antitriplets [2] is attractive and will be considered. In this approach the horizontal hierarchy hypothesis (HHH) is reasonable. According to HHH, the structure of fermion mass matrices reflects the pattern of horizontal symmetry breaking and the mass hierarchy between families is related to a definite hierarchy in this breaking. The simplest realization of HHH provides that the quark and lepton masses are induced in a "see–saw" manner [2] due to their mixing with some additional superheavy fermions.

[1]e-mail: edward@lep1.tau.ac.il

Let us consider the main elements of BFS, to be interpreted in the framework of an 8-dimensional Kaluza geometry [1] (a detailed description of BFS is contained in [2]).

(a) The fermion content.

Quarks and leptons are contained in the following representations of $SU(2) \bigotimes U(1) \bigotimes SU(3)_H$:

$$f_{L\alpha_f}: \quad \begin{pmatrix} u \\ d \end{pmatrix}_{L\alpha_f} (2, \tfrac{1}{3}, 3), \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_{L\alpha_f} (2, -1, 3);$$
$$f_R^{\alpha_f}: \quad u_R^{\alpha_f}(1, \frac{4}{3}, \tilde{3}), \qquad d_R^{\alpha_f}(1, \frac{2}{3}, \tilde{3}), \quad e_R^{\alpha_f}(1, -2, \tilde{3}) \tag{1}$$

where we retain the family ($SU(3)_H$) index: $\alpha_f = 1, 2, 3$. Let us denote the charged fermions of (1) by q. The additional superheavy fermions have the following form:

$$F_L^{\alpha_f}: \quad U_L^{\alpha_f}(1, \frac{4}{3}, \tilde{3}); \qquad D_L^{\alpha_f}(1, -\frac{2}{3}, \tilde{3}); \quad E_L^{\alpha_f}(1, -2, \tilde{3})$$
$$F_{R\alpha_f}: \quad U_{R\alpha_f}(1, 4/3, 3); \quad D_{R\alpha_f}(1, -2/3, 3); \quad E_{R\alpha_f}(1, -2, 3); \quad N_{R\alpha_f}(1, 0, 3) \tag{2}$$

Let us denote the charged fermions of (2) by Q.

(b) The Dirac see–saw mechanism.

The most general Yukawa couplings allowed by the symmetry are

$$g_{nF} \bar{Q}_{R\alpha_f} Q_L^{\beta_f} \xi^{(n)}_{\alpha_f \beta_f} + h.c., \tag{3}$$
$$g_f \bar{q}_{L\alpha_f} Q_{R\alpha_f} \phi^0 + h.c.,$$
$$\mu_f \bar{Q}_L^{\alpha_f} q_R^{\alpha_f} + h.c. \tag{4}$$

Here ϕ^0 is the neutral component of the standard $SU(2) * U(1)$ Higgs doublet $(2, -1, 1)$ ($\langle \phi^0 \rangle = v = (\sqrt{8} G_F) = 250\,\mathrm{GeV}$), η is a real $SU(2) * U(1) * SU(3)_H$

singlet scalar ($\langle\eta\rangle = \mu/G_\eta$) and $\xi^{(n)}$, $n = 0, 1, 2$. The main vacuum expectation values (VEV) of (ξ) are developed to the components $\langle\xi^{(0)}_{33}\rangle = r_3$, $\langle\xi^{(1)}_{23}\rangle = p_2$, $\langle\xi^{(2)}_{12}\rangle = p_1$ (see the matrix of VEV in [2]) with the natural (about 5–10 fold) hierarchy $r_3 >> p_2 >> p_1$. Inserting the VEVs into the couplings (3) and (4), one obtains the full 6×6 fermions mass matrices [2].

(c) Charged and neutral currents.

$$\begin{array}{ll} \frac{1}{2}g_H H^{(A)}_\mu[\bar L\lambda(A)\gamma_\mu L - \bar R\lambda^T(A)\gamma_\mu R] & \text{for}\quad q; \\ \frac{1}{2}g_H H^{(A)}_\mu[\bar R\lambda(A)\gamma_\mu R - \bar L\lambda^T(A)\gamma_\mu L] & \text{for}\quad Q. \end{array} \tag{5}$$

Here $H^{(A)}_\mu$ are gauge bosons, $A = 1..8$.

A geometrization of (a), (b) and (c) can be provided in the framework of an 8–dimensional space-time manifold with torsion and external spinor matter.

Let us summarize the main results of $M^4\times S^1\times S^1\times S^1\times S^1$ interpretations of BFS for the quark sector. We choose the 8–dimensional hyperdensity of the Lagrangian in the form [1]

$$L = \frac{\sqrt{-^8G^0}}{4}[\frac{1}{\kappa}(\tilde\Lambda -^8\tilde R^0) + ihc\chi^\mu\bar\Psi\Gamma^{0M}\nabla^0_M\Psi] \tag{6}$$

where ${}^8G^0$ is the determinant of the 8–dimensional metric $(+\ -\ -\ -\ -\ -\ -\ -)$; ${}^8\tilde R^0$ is the scalar curvature with torsion, ${}^8\tilde R^0 = {}^8R^0 - {}^8S^0_{MNP}{}^8S^{0MNP}$, ${}^8S^0_{MNP}$ is the torsion tensor ($M, N, P = 1..8$); Ψ is a 16-component spinor in the 8-dimensional manifold; $\nabla^0_M\Psi$ is an 8-dimensional covariant derivative of a spinor; $\kappa = 8\pi k/c^4$, $k = G$. Γ^0_M are the Dirac 16-matrices which form a Clifford algebra $C(1,7)$ with real spinors. $\tilde\Lambda$ is the effective cosmological constant.

The procedures of geometrization:

1. A $1+1+1+1+4$ splitting of the 8-dimensional manifold is provided by

   $${}^8G_{MN} = g_{MN} - \lambda_M\lambda_N - \sigma_M\sigma_N - \omega_M\omega_N - \theta_M\theta_N$$

   (from the 8–dimensional metric the neutral vector fields, which correspond to λ^3 and λ^8, are obtained).

2. A conformal transformation (from the conformal factor the necessary scalars are obtained).

3. An essential use of torsion (the torsion produces charged bosons and the see–saw mechanism)

4. We postulate a special dependence [1] of all values on additional coordinates and averaging of the Lagrangian density over the small periods of the additional coordinates.

5. We use a special representation of the 8-dimensional spinors by standard 4-component functions of fermions for 3 generations and 2 levels (q and Q).

In our consideration x^7 and x^8 bear the responsibility for horizontal currents. Accordingly we choose the neutral vector bosons along to the monads of the 7th and 8th coordinates. Thus for (5) we have the following dependence of quarks on the additional coordinates:

$$\begin{aligned}(q;Q) = (q;Q)(x^\mu)\exp(i\alpha x^5\varepsilon_5 + i\beta x^6\varepsilon_6 \\ + i\gamma x^7\varepsilon_7 + i\delta x^8\varepsilon_8)\end{aligned} \tag{7}$$

where the marks of the generations have the form

$$\begin{array}{l} \varepsilon_5^{\alpha_f} = \varepsilon_6^{\alpha_f} = 0;\quad \varepsilon_7^1 = 1;\quad \varepsilon_8^1 = 1;\quad \varepsilon_7^2 = -1; \\ \quad \varepsilon_8^2 = 1;\quad \varepsilon_7^3 = 0;\quad \varepsilon_8^3 = -2 \qquad \text{for}\quad q_L,\ Q_R; \\ \varepsilon_5^{\alpha_f} = \varepsilon_6^{\alpha_f} = 0;\quad \varepsilon_7^1 = -1;\quad \varepsilon_8^1 = -1;\quad \varepsilon_7^2 = 1; \\ \varepsilon_8^2 = -1;\quad \varepsilon_7^3 = 0;\quad \varepsilon_8^3 = 2 \qquad \text{for}\quad q_R,\ Q_L; \end{array} \tag{8}$$

$\alpha_f = 1..3$, $Q_3 = 1/2(\varepsilon_7+\varepsilon_8)$, $Q_8 = 1/2(\varepsilon_7-\varepsilon_8)$. The neutral vector fields for Q_3 and Q_8 have the form

$$\begin{aligned} H(3)_\mu &= \pm\frac{c^2}{4\sqrt{2k}}(\omega_\alpha + \theta_\alpha); \\ H(8)_\mu &= \pm\frac{c^2}{4\sqrt{2k}}(\omega_\alpha - \theta_\alpha) \end{aligned} \tag{9}$$

with the gauge constant

$$\tilde g_H = \alpha\frac{4\sqrt{2k}}{c^2}(hc)\omega^7 \qquad (\alpha = \beta = \gamma = \delta).$$

Let us classify the quarks by sets of integer harmonics with the following electric charge assignment:

$$\begin{array}{lll} Q_{em} = 1/3(\varepsilon_5 + \varepsilon_6) = 2/3 & \text{for} & \text{“}up''; \\ Q_{em} = 1/3(\varepsilon_5 + \varepsilon_6) = -1/3 & \text{for} & \text{“}down''. \end{array}$$

Each set of harmonics can be represented as a sum of the following contributions:

1. Marks of generations (7).

2. Marks of mass level

   $$\begin{array}{lll} \varepsilon_5^{\alpha^f} = 1;\quad \varepsilon_6^{\alpha^f} = \varepsilon_7^{\alpha^f} = \varepsilon_8^{\alpha^f} = 0 & \text{for} & q; \\ \varepsilon_5^{\alpha^f} = \varepsilon_7^{\alpha^f} = \varepsilon_8^{\alpha^f} = 0;\quad \varepsilon_6^{\alpha^f} = 1 & \text{for} & Q. \end{array}$$

3. Marks of horizon

   $$\begin{array}{lll} \varepsilon_5^{\alpha^f} = \varepsilon_7^{\alpha^f} = \varepsilon_8^{\alpha^f} = 0;\quad \varepsilon_6^{\alpha^f} = 1 & \text{for} & \text{“}up''; \\ \varepsilon_5^{\alpha^f} = -1;\quad \varepsilon_6^{\alpha^f} = -1; & & \\ \quad \varepsilon_7^{\alpha^f} = \varepsilon_8^{\alpha^f} = 0 & \text{for} & \text{“}down''. \end{array}$$

The 16–component spinor is constructed from 4–component spinors wave functions:

$$\Psi = \begin{pmatrix} u \\ d \\ U \\ D \end{pmatrix} = \begin{pmatrix} \psi(1)_L + \psi(1)_R \\ \psi(2)_L + \psi(2)_R \\ \psi(3)_L + \psi(3)_R \\ \psi(4)_L + \psi(4)_R \end{pmatrix} \tag{10}$$

For the interpretation of see-saw (3) and (4) we must suppose that the crossing of harmonics takes place [1]:

$$\psi_L(s) = a_{ij}(q_{jL}; Q_{jL});$$
$$\psi_R(s) = b_{ij}(q_{jR}; Q_{jR});$$
$$\psi_{L(R)}(S) = a(b)_{ij}(q_{jL(R)}; Q_{jL(R)}) \exp(i\alpha \sum_{i=5}^{8} \varepsilon_i^{L(R)} x^i) + z_{j0}\eta_{j0} f_{q_s} \exp(i\alpha \sum_{i=5}^{8} \varepsilon_i^{R(L)} x^i) \quad (11)$$

Here the additional harmonics to the Higgs ones are different from the basic harmonics. Let us find the absolute values of harmonic differences between Q_R and Q_L for every α_f and β_f. We will present the result in the matrix form

$$\begin{pmatrix} 2\gamma x^7 + 2\delta x^8 & 2\delta x^8 & \gamma x^7 - \delta x^8 \\ 2\delta x^8 & -2\gamma x^7 + 2\delta x^8 & -\gamma x^7 - \delta x^8 \\ \gamma x^7 - \delta x^8 & -\gamma x^7 - \delta x^8 & -4\delta x^8 \end{pmatrix}. \quad (12)$$

Due to this metric analogue of the mass matrix of fermions, we can obtain every pattern of the mass matrix of quarks. The connections (3) are obtained from the mass term of hyperdensity, namely, from the terms with the derivatives including $\Gamma(5)$ and $\Gamma(6)$ (see [1]). The connections (4) arise due to the part in the mass term of hyperdensity which contains torsion, namely, due to the terms with

$$S(876) = S^8_{MNP}\theta^M \omega^N \sigma^P;$$
$$S(875) = S^8_{MNP}\theta^M \omega^N \sigma^P. \quad (13)$$

In this case we must introduce a crossing of harmonics between $q_{L(R)}$ and $Q_{R(L)}$ for identical generations. The difference of harmonics is in this case $\alpha x^5 - \beta x^6$. After the averaging over all additional coordinates, the connections, absent in the see–saw mechanism, disappear. According to the scheme of crossing harmonics, the conformal factor must have the form [1]

$$\begin{aligned} \chi = 1 \; + \; & t_0[\phi_0 \exp(i(\alpha x^5 - \beta x^6)) - h.c.] \\ & + h_0[\xi_0 \exp(-i4\delta x^8) - h.c.] \\ & + h_1[\xi_1 \exp(-i(\gamma x^7 + \delta x^8)) - h.c.] \\ & + h_2[\xi_2 \exp(2i\delta x^8) - h.c.]. \end{aligned} \quad (14)$$

This form of the conformal factor provides the general structure of the VEV matrix in BFS. Due to the presence of the constant part

$$\xi_0 = \xi_0' + r_3; \qquad \xi_1 = \xi_1' + p_2; \qquad \xi_2 = \xi_2' + p_1,$$

we can reproduce the hierarchy of VEV and quark masses, just as is done in the BFS. We will not discuss the charged vector bosons because it is a sufficiently standard procedure and it has been described in [1].

Thus in the framework of the Kaluza approach we can describe the main elements of broken family symmetry without a neutral sector of fermions and Majorana see–saw mechanism [2]. This fact testifies that the ideas of Ref. [1] are sufficiently useful for more complicated particle theories.

References

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[2] M.Yu. Khlopov and A.S. Sakharov A.S., a paper in this issue, p. 164–176.