

ON $SL(3, \mathbb{C})$ -COVARIANT SPINOR EQUATION AND GENERALIZED DUFFIN-KEMMER ALGEBRA

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The $SL(3, \mathbb{C})$ -covariant 9-dimensional equation for a free 3-spinor particle is transformed into the Dirac-like form: $(p_A \delta^A - M)\Psi = 0$. However, the corresponding δ matrices do not satisfy the Dirac algebra. It is shown that δ^A lead to a Finslerian generalization of the Duffin-Kemmer algebra. The Appendix contains an explicit realization of the δ matrices.

During several years, a theory called binary geometro-physics is successfully developed [1]. Within the framework of this theory, in the article [2], the concept of a 3-spinor as a tensor on the complex vector space \mathbb{C}^3 with the trilinear antisymmetric “scalar multiplication” $\mathbb{C}^3 \times \mathbb{C}^3 \times \mathbb{C}^3 \rightarrow \mathbb{C}$ was introduced. Isometries of such a space form a group isomorphic to the group $SL(3, \mathbb{C})$ of unimodular complex 3×3 -matrices. In the same article, a homomorphism of $SL(3, \mathbb{C})$ into the isometry group of a 9-dimensional (over \mathbb{R}) flat Finslerian space with the metric

$$\begin{aligned} G^{ABC} p_A p_B p_C &= (p_0^2 - p_1^2 - p_2^2 - p_3^2) p_8 \\ &- (p_4^2 + p_5^2 + p_6^2 + p_7^2) p_0 + 2(p_4 p_6 + p_5 p_7) p_1 \\ &+ 2(p_5 p_6 - p_4 p_7) p_2 + (p_4^2 + p_5^2 - p_6^2 - p_7^2) p_3 \end{aligned} \quad (1)$$

was constructed. Here $A, B, C = \overline{0, 8}$ and p_A is a real 9-vector. It should be noted that (1) passes into the ordinary pseudo-Euclidean 4-metric $g^{\mu\nu} p_\mu p_\nu = p_0^2 - p_1^2 - p_2^2 - p_3^2$ after the procedure of the dimensional reduction. From the physical point of view, the expression (1) is the “scalar cube” of a vector in the 9-dimensional momentum space.

Let i^r and $\beta_{\dot{s}}$ ($r, s = \overline{1, 3}$) be 3-spinors of rank one, while $P^{r\dot{s}}$ be a 3-spinor of rank two, such that the matrix $P \equiv \|P^{r\dot{s}}\|$ is Hermitian. (As ever, the dotted index means that the corresponding quantity is transformed according to the complex conjugate representation of the group $SL(3, \mathbb{C})$.) It is not difficult to show that

$$\det P = G^{ABC} p_A p_B p_C \quad (2)$$

when the relations between $P^{r\dot{s}}$ and p_A have the form

$$\left. \begin{aligned} P^{1\dot{1}} &= p_0 + p_3, & P^{1\dot{2}} &= p_1 - ip_2, & P^{1\dot{3}} &= p_4 - ip_5 \\ P^{2\dot{1}} &= p_1 + ip_2, & P^{2\dot{2}} &= p_0 - p_3, & P^{2\dot{3}} &= p_6 - ip_7 \\ P^{3\dot{1}} &= p_4 + ip_5, & P^{3\dot{2}} &= p_6 + ip_7, & P^{3\dot{3}} &= p_8 \end{aligned} \right\} \quad (3)$$

In Ref. [3], to describe a free 3-spinor particle with a 9-momentum p_A , the $SL(3, \mathbb{C})$ -covariant equation

$$\left. \begin{aligned} P^{r\dot{s}} \beta_{\dot{s}} &= M i^r \\ P_{r\dot{s}} i^r &= M^2 \beta_{\dot{s}} \end{aligned} \right\} \quad (4)$$

was proposed, where $P^{r\dot{s}}$ are defined by (3), M is a real scalar, and $P_{r\dot{s}}$ are the cofactors of $P^{r\dot{s}}$. It is natural to call M the 9-mass of a particle because substituting the upper equality of (4) into the lower one (and vice versa) gives a Finslerian analog of the Klein-Gordon equation for each 3-spinor component: $(G^{ABC} p_A p_B p_C - M^3) i^r = 0$, $(G^{ABC} p_A p_B p_C - M^3) \beta_{\dot{s}} = 0$. In [3], it was also shown that (4) split into the standard 4-dimensional Dirac and Klein-Gordon equations after the group reduction $SL(3, \mathbb{C}) \rightarrow SL(2, \mathbb{C})$.

In general, Eq. (4) is quadratic with respect to p_A . This follows from (3) and the fact that $P_{r\dot{s}}$ are proportional to 2×2 -minors of the matrix P . The purpose of the present paper is to transform (4) into the form of an equation linear with respect to the 9-momentum p_A . To this end, it is possible to use the method of Duffin and Kemmer [4, 5].

Let us introduce the new variables $\xi_1, \xi_2, \dots, \xi_6$ such that

$$\left. \begin{aligned} P^{2\dot{1}} i^1 - P^{1\dot{1}} i^2 &= M \xi_1, & P^{2\dot{2}} i^1 - P^{1\dot{2}} i^2 &= M \xi_4 \\ P^{3\dot{1}} i^1 - P^{1\dot{1}} i^3 &= M \xi_2, & P^{3\dot{2}} i^1 - P^{1\dot{2}} i^3 &= M \xi_5 \\ P^{3\dot{1}} i^2 - P^{2\dot{1}} i^3 &= M \xi_3, & P^{3\dot{2}} i^2 - P^{2\dot{2}} i^3 &= M \xi_6 \end{aligned} \right\} \quad (5)$$

With the help of (5), one can rewrite the lower equality of (4) in the following form:

$$\left. \begin{aligned} P^{3\dot{3}} \xi_4 - P^{2\dot{3}} \xi_5 + P^{1\dot{3}} \xi_6 &= M \beta_1 \\ -P^{3\dot{3}} \xi_1 + P^{2\dot{3}} \xi_2 - P^{1\dot{3}} \xi_3 &= M \beta_2 \\ -P^{3\dot{1}} \xi_4 + P^{2\dot{1}} \xi_5 - P^{1\dot{1}} \xi_6 &= M \beta_3 \end{aligned} \right\} \quad (6)$$

Thus (4) is equivalent to the set of equations $P^{r\dot{s}} \beta_{\dot{s}} =$

