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PREFERRED BASIS IN THE MANY-WORLDS INTERPRETATION IN QUANTUM THEORY AND THE SYMMETRIES OF THE SYSTEM

A.O. Barvinsky†,‡ and A.Yu. Kamenshchik‡

† P.N. Lebedev Physical Institute, Russian Academy of Sciences, 53 Leninsky Prospect, Moscow 117924, Russia

‡ Nuclear Safety Institute, Russian Academy of Sciences, 52 Bolshaya Tulskaya St., Moscow 113191, Russia

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We apply a recently developed proposal for the choice of a preferred basis in the many-worlds interpretation of quantum mechanics and quantum cosmology to systems possessing some non-Abelian symmetry. It is shown that in this case the system subdivision into subsystems implying the choice of the preferred basis should be arranged in such a way that the wave functions of the subsystems belong to the trivial representation of the corresponding non-Abelian group. Otherwise the preferred basis choosing procedure does not work.

1. Introduction

The development of quantum cosmology in recent years [1] served as an impact for studying the conceptual foundations of quantum mechanics. The idea of the quantum birth of the Universe [2–5] is naturally combined with the treatment of the whole Universe as a quantum object, which in another context was put forward in the frames of the many-worlds interpretation (MWI) of quantum mechanics (QM) [6–9] pioneered in 1957 by Everett [6]. In our preceding papers [10, 11] we have studied the problem of the choice of a preferred basis in the MWI of QM which was also widely discussed in Ref. [12–18] and is closely related to the problem of explaining the classical behaviour of the Universe.

It is well-known that in the framework of the so-called Copenhagen interpretation of QM [19, 20] there are two fundamental processes: unitary evolution of the wave function according to the Schrödinger equation and the so-called wave packet reduction in the process of quantum measurement [20]. The meaning of the latter consists in the abrupt and non-causal elimination of the part of the wave function that corresponds to those possible outcomes of quantum measurement that were not realized. This phenomenon is usually explained by a special role of a classical device in quantum measurement or even by a special role of consciousness in our world, but in any case this postulate restricts the "sovereignty" of the quantum-mechanical description of reality.

This situation can be regarded unsatisfactory from the point of view of those who, like H. Everett, are guided by a desire to reduce to a minimum the number of fundamental principles of the theory. Moreover, trying to apply QM to the consideration of cosmological problems we stumble upon the impossibility of finding a place for a classical realm or an external classical observer. These two groups of reasons stimulated the development of the MWI of QM. The main idea of this interpretation is a priority of QM over classical mechanics and the belief in the objective nature of the wave function. Thus in the frame of the MWI there is only one object describing the physical reality — the wave function – and only one dynamical process — the Schrödinger evolution. The wave packet reduction postulate is given up. Instead we can speak of a simultaneous realization of different outcomes of quantum measurement in different "branches" of the wave function of the Universe or in different "parallel" Everett worlds. This approach to the problems of quantum cosmology seems to be promising and we shall attempt to be consistent in carrying it out in this paper.

Resolving some old problems of interpretation of quantum theory, the MWI invokes new questions. A special place between them occupies the problem of the choice of a preferred basis or, in other words, of the procedure determining which specific pieces of the wave function of the Universe can be regarded as those describing autonomous Everett worlds or branches. In our preceding papers [10, 11] we have revived the old proposal of Zeh [12] and gave somewhat different and more transparent arguments in favour of this proposal. It is also necessary to stress that this basis, which we call bi-orthogonal, was first introduced at the dawn of QM (and many years before the MWI came into existence) by Schrödinger for the purpose of describing quantum correlations between interacting subsystems [21]. An similar basis was used also by Schmidt in a purely mathematical context [22].

Here we continue to study the properties of the bi-orthogonal basis. We study the relation between the preferred basis choosing procedure and the symmetry of the system under consideration. It is shown that in case the system possesses some non-Abelian symmetry, its subdivision into subsystems implying the choice of the preferred basis should be arranged in such a way that the wave functions of the subsystems belong to the trivial representation of the corresponding non-Abelian group. In the opposite case the preferred basis choosing procedure does not work.

The paper has the following structure: in Sec. 2 we recapitulate the procedure of choosing a preferred basis developed in Refs. [10, 11] and in Sec. 3 we apply our prescription to a system possessing a non-Abelian symmetry.

2. The choice of a preferred basis

The MWI of QM resolves some problems and paradoxes arising in other interpretations. At the same time it gives rise to some additional problems. An especially important problem among them is the choice of a preferred basis. To explain the essence of this problem, it makes sense to remind the wave function "branching" mechanism in measurement-like processes. For illustrative purposes we shall consider the well-known example of the Stern-Gerlach experiment.

Let the system consist of a device which is initially in a state $|\Phi\rangle_0$ and is ready to measure the spin z-component of an atom, which is in the state

$$(c_1|\uparrow\rangle + c_2|\downarrow\rangle),$$

where $|\uparrow\rangle$ and $|\downarrow\rangle$ are $s_z = \pm\frac{1}{2}$ orthonormal eigenstates of an atom. The initial state of the system as a whole is

$$|\Psi\rangle_{\rm in} = (c_1|\uparrow\rangle + c_2|\downarrow\rangle)|\Phi\rangle_0. \tag{2.1}$$

We see that before the measurement both the device and the atom are in pure states. Let us now introduce the unitary operator $\hat{U}$ describing the interaction between the atom and the device. The measurement operator $\hat{U}$ satisfies the following rule [21]:

$$\hat{U}|\uparrow\rangle|\Phi\rangle_0 = |\uparrow\rangle|\Phi_\uparrow\rangle, \quad \hat{U}|\downarrow\rangle|\Phi\rangle_0 = |\downarrow\rangle|\Phi_\downarrow\rangle \tag{2.2}$$

where $|\Phi_\uparrow\rangle$ and $|\Phi_\downarrow\rangle$ are orthonormal eigenstates of the device indicating the fact of measuring respectively the values $s_z = \frac{1}{2}$ and $s_z = -\frac{1}{2}$. Under the action of $\hat{U}$ the state (2.1) transforms into

$$|\Psi\rangle_{\rm out} = \hat{U}|\Psi\rangle_{\rm in} = c_1|\uparrow\rangle|\Phi_\uparrow\rangle + c_2|\downarrow\rangle|\Phi_\downarrow\rangle. \tag{2.3}$$

From the point of view of the Copenhagen interpretation, a unitary transition from the state (2.1) to the state (2.3) is only the first part of the process of quantum measurement. We know that we can actually measure only one value of s_z, but in (2.3) there are two terms corresponding to both possible results of measurement. The second part of the quantum measurement consists in eliminating one of these terms:

$$|\Psi\rangle_{\rm out} \Rightarrow |\uparrow\rangle|\Phi_\uparrow\rangle \quad \text{or} \quad |\Psi\rangle_{\rm out} \Rightarrow |\downarrow\rangle|\Phi_\downarrow\rangle. \tag{2.4}$$

The process (2.4) is nothing else but a reduction of the wave packet. However, in the framework of the MWI we reject the process (2.4) and recognize the simultaneous existence of both terms in the superposition (2.3): the measurement process reduces to the arrangement of correlations between the atom and the device and both outcomes of the experiment exist in parallel worlds. Thus, instead of the wave packet reduction, we have a "branching" of our world. From the mathematical point of view, the branching is merely defactorization of the wave function of a system corresponding to a subdivision of this system into subsystems (see Eq. (2.3)) and these subsystems undergo the transition from pure quantum states to mixed ones.

Generally speaking, each transition of the wave function of a system consisting of two subsystems from the factorized state

$$|\Psi\rangle_{\rm in} = |\varphi\rangle|\chi\rangle \tag{2.5}$$

to the defactorized state

$$|\Psi\rangle_{\rm out} = \sum_{i=1}^{n} |\varphi_i\rangle|\chi_i\rangle, \tag{2.6}$$

where the states $|\varphi\rangle$ and $|\varphi_i\rangle$ describe one subsystem and the states $|\chi\rangle, |\chi_i\rangle$ the other one, and $n > 1$ can be regarded as a process of branching or defactorization of the wave function.

We see that in the decomposition (2.6) each state of one subsystem $|\varphi_i\rangle$ has a uniquely determined counterpart $|\chi_i\rangle$, which is usually called the "relative state" [6]. However, at the same time, we can choose another set of basis states $|\tilde{\varphi}_i\rangle$ instead of $|\varphi_i\rangle$. In this case we shall have another set of relative states $|\tilde{\chi}_i\rangle$ and the wave function (2.6) will be written in the following form:

$$|\Psi\rangle_{\rm out} = \sum_{i=1}^{n} |\tilde{\varphi}_i\rangle|\tilde{\chi}_i\rangle. \tag{2.7}$$

From the mathematical point of view, there is no essential difference between the formulas (2.6) and (2.7), they simply describe the same wave function, but written in different bases. But from the physical point of view a choice of different bases implies different decompositions of the wave function into sets of branches which correspond to different real worlds, able to be perceived experimentally. Thus, we have to make a definite choice of a certain preferred decomposition or preferred basis.

One can say that within the usual "common sense" it is impossible to choose, in the case of Stern-Gerlach experiment, a basis other than

$$|\uparrow\rangle|\Phi_\uparrow\rangle \quad \text{and} \quad |\downarrow\rangle|\Phi_\downarrow\rangle, \tag{2.8}$$

because our device is constructed and arranged to measure just the spin z-component of an atom. It is true, but in order to separate other possible choices of the basis we have to rely on some classical properties of the device. However, this deprives us of the main advantage of the MWI — its purely quantum nature. Moreover, common sense is applicable only in simple quantum-mechanical experiments. In the more complicated case of quantum cosmology common sense does not always work. The point is that it is necessary to work out a procedure of constructing the preferred basis, originating from purely quantum notions. We can try to extract this procedure from a treatment of rather simple quantum-mechanical problems, generalize it and then apply it to more general and complicated situations amounting to problems of quantum cosmology.

The preferred basis construction which we would like to advocate consists of two steps: (i) splitting of the system under consideration into certain subsystems and (ii) choosing the proper basis for one subsystem and the basis of relative states for the other. It is necessary to stress that, from the point of view of our prescription, we can choose any subdivision of a system into subsystems, because for every such subdivision our prescription gives a unique preferred basis. In contrast to the Copenhagen interpretation, where observation and measurement play a special role, in the MWI they are simply interactions between subsystems. Therefore, we shall treat our subsystems on an equal footing without indication which of them is the observer and which is the observable. All notations concerning the corresponding subsystem will be marked by the subscripts I and II.

It seems rather reasonable to require that the belonging of different terms of the wave function decomposition to different worlds must be accompanied by their mutual orthogonality; moreover, we require also orthonormality of substates corresponding to our subdivision of a system into subsystems. It leads immediately to the following decomposition:

$$|\Psi\rangle = \sum_n c_n |n\rangle_I |n\rangle_{II}, \tag{2.9}$$

where both sets of basis vectors $|n\rangle_I$ and their relative states $|n\rangle_{II}$ are orthonormal

$${}_I\langle n|m\rangle_I = \delta_{nm}, \quad {}_{II}\langle n|m\rangle_{II} = \delta_{nm} \tag{2.10}$$

and c_n are some complex coefficients which determine a priori probabilities p_n of realizing the nth Everett world:

$$p_n = |c_n|^2, \qquad \sum_n |c_n|^2 = \langle\Psi|\Psi\rangle = 1. \tag{2.11}$$

This prescription for chooosing a preferred basis yields also a constructive algorithm for finding it. Consider the density matrix of a total system $\hat\rho = |\Psi\rangle\langle\Psi|$ and the density matrix of the first subsystem I, obtained by tracing out the degrees of freedom of the subsystem II

$$\hat\rho_I = \mathrm{Tr}_{\,II} |\Psi\rangle\langle\Psi|. \tag{2.12}$$

Substituting the decomposition (2.9) and using the orthonormality condition (2.10), one finds that the latter has the form

$$\hat\rho_I = \sum_n |c_n|^2 |n\rangle_I \, {}_I\langle n| \tag{2.13}$$

whence one concludes that the vectors $|n\rangle_I$ of the preferred basis solve the eigenvalue problem for the density matrix $\hat\rho_I$:

$$\hat\rho_I |n\rangle_I = p_n |n\rangle_I, \qquad p_n = |c_n|^2 \tag{2.14}$$

with the eigenvalues exactly coinciding with the probability weights (2.11) of the Everett worlds. Similarly to $|n\rangle_I$, the basis vectors $|n\rangle_{II}$ are the eigenvectors of the density matrix of the second subsystem:

$$\hat\rho_{II} = \mathrm{Tr}_{\,I} |\Psi\rangle\langle\Psi|, \quad \hat\rho|n\rangle_{II} = p_n |n\rangle_{II}. \tag{2.15}$$

Both density matrices $\hat\rho_I$ and $\hat\rho_{II}$ are Hermitian, positive semi-definite and (in view of the normalizability of $|\Psi\rangle$) bounded operators. Therefore they possess a denumerable eigenvalue spectrum and their orthonormal eigenvectors are unique up to inessential phase factors. The only exceptions are the invariant subspaces of $\hat\rho_I$ and $\hat\rho_{II}$ corresponding to possible degenerate eigenvalues, whose eigenvectors can be determined up to unitary rotations. These subspaces can be only finite-dimensional because the sum of eigenvalues over each subspace is less than (or equal to) 1 in view of (2.11).

The basis (2.9), obtained by reasoning of the above type, coincides with Schrödinger's bi-orthogonal basis [21], introduced for studying quantum correlations between interacting quantum systems, or with the "Schmidt canonical basis" of Zeh [12], (this name originates from the classical paper of Schmidt [22] on integral equations), used in the context of the MWI. This basis can be obtained by solving two eigenvalue problems (2.13) and (2.14) for the density matrices of relevant subsystems. The non-uniqueness of this solution in the invariant subspaces of the degenerate eigenvalues was also mentioned in Ref. [12], but with no concrete proposal for its resolution. Here we will go somewhat further and suggest an additional principle in order to fix this basis uniquely. From Eqs. (2.13)–(2.14) it follows that the preferred basis depends on the quantum state $|\Psi\rangle$ of the total system and consequently evolves in accordance with the Schrödinger evolution of $|\Psi\rangle = |\Psi(t)\rangle$. Then it seems natural to require that the decomposition of these invariant Hilbert subspaces into equally probable Everett worlds be stable against this dynamical evolution. To demonstrate the efficiency of this principle we return to the above example of the Stern-Gerlach experiment.

The density matrices of the measurement device and the atom, corresponding to the whole system's state after measurement (2.3), take the form

$$\hat\rho_{\mathrm{device}} = |c_1|^2 |\Phi_\uparrow\rangle\langle\Phi_\uparrow| + |c_2|^2 |\Phi_\downarrow\rangle\langle\Phi_\downarrow|$$

$$\hat{\rho}_{\rm atom} = |c_1|^2|\uparrow\rangle\langle\uparrow| + |c_2|^2|\downarrow\rangle\langle\downarrow|$$

and according to our algorithm the preferred basis is given in the non-degenerate case $|c_1|^2 \neq |c_2|^2$ by the vectors (2.4).

For $|c_1|^2 = |c_2|^2 = \frac{1}{2}$ this procedure becomes insufficient because the preferred basis in our two-dimensional Hilbert space is now not unique. For example, instead of (2.4) one can take the basis of vectors $|\rightarrow\rangle|\Phi_\rightarrow\rangle$ and $|\leftarrow\rangle|\Phi_\leftarrow\rangle$, where

$$\begin{aligned}
|\rightarrow\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle), \\
|\leftarrow\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle), \\
|\Phi_\rightarrow\rangle &= \frac{1}{\sqrt{2}}(|\Phi_\uparrow\rangle + |\Phi_\downarrow\rangle), \\
|\Phi_\leftarrow\rangle &= \frac{1}{\sqrt{2}}(|\Phi_\uparrow\rangle - |\Phi_\downarrow\rangle).
\end{aligned} \tag{2.16}$$

For the case $c_1 = c_2 = 1/\sqrt{2}$ Eq. (2.15) also solves the eigenvalue problems. The new basis is, however, unstable with respect to the measurement-like interaction between the device and the atom. Indeed, two branches of $|\Psi\rangle$ in this basis are

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|\rightarrow\rangle|\Phi_\rightarrow\rangle + \frac{1}{\sqrt{2}}|\leftarrow\rangle|\Phi_\leftarrow\rangle. \tag{2.17}$$

In contrast to the branches (2.15) of the decomposition (2.16), the branches (2.4) of the decomposition (2.3) are not destroyed during the dynamical evolution by the operator $\hat{U}$ and thus form the unique preferred basis giving a reasonable many-worlds picture of the Stern-Gerlach experiment.

3. Preferred basis and symmetries of the system

The preferred basis choosing procedure considered in Sec. 2 is constructed in such a manner as to be independent of a particular splitting of the system into subsystems. However, in some cases the requirement of definiteness and uniqueness of our choice can affect the procedure of splitting. We can present an example of such a "back reaction" of the prescription for preferred basis on the splitting.

Let us assume that there is some conserved non-Abelian symmetry in our Universe, i.e., the Hamiltonian governing the Universe evolution contains operators belonging to some non-trivial representations of a non-Abelian group in such a way that this Hamiltonian as a whole belongs to a singlet representation of this group. Besides, we suppose that the wave function of the Universe belongs to a singlet representation of the symmetry group as well. Under a singlet representation of some group one usually means the trivial representation of this group. Speaking of a wave function in a singlet (trivial) representation, we mean that this wave function is intact under the action of all group elements, $g|\Psi\rangle = |\Psi\rangle$, and is annihilated by each generator of the corresponding Lie algebra. In the case of an operator belonging to a singlet representation (specifically, we consider a Hamiltonian) the following equality is correct:

$$gHg^{-1} = H$$

where g is an arbitrary group element. In other terms, it means that the Hamiltonian commutes with all generators of the corresponding Lie algebra. It is necessary to stress that the wave function can be constructed in such a way that, being in a singlet (trivial) state, it can contain components belonging to non-singlet (nontrivial) states. A mathematical background of this phenomenon consists in the fact that direct products of elements belonging to non-trivial representations of a non-Abelian group can contain the trivial representation in the decomposition into a direct sum. For example a couple of spin-1/2 particles can have a total spin equal to zero. Analogously, a baryon consisting of three colour quarks belonging to a non-trivial (fundamental) representation of the $SU(3)$ group can be colourless and belong to the trivial representation of this group. In accordance with what is written, we can choose the splitting of the system under consideration (the Universe) in such a way that both subsystems are in non-singlet states, while the Universe as a whole is in a singlet state. All the subsequent argument is valid for an arbitrary gauge theory with non-Abelian symmetry, but to make our picture clear we shall speak of the colour group $SU(3)$ in quantum chromodynamics and take as a subsystem with a non-zero colour charge a single quark or another colour combination of quarks and antiquarks. Then one subsystem is that quark and the other is the rest of the Universe. The wave function takes the following form:

$$|\Psi\rangle = \sum_{i=1}^{3} \frac{1}{\sqrt{3}}|\varphi_i\rangle|\chi_i\rangle. \tag{3.1}$$

Here the three functions $|\varphi_i\rangle$ belong to the fundamental representation of the $SU(3)$ group, while the functions $|\chi_i\rangle$ belong to a representation contragradient to the fundamental one. The density matrices for both subsystems have a degenerate spectrum ($p_i = 1/3$ for $i = 1, 2, 3$). Thus, in order to get a unique preferred basis, we must apply the principle of dynamical stability of this basis (see Sec. 2). The Hamiltonian governing the evolution of the wave function (3.1) can be presented in the form

$$H = H_\varphi \bigotimes I_\chi + I_\varphi \bigotimes H_\chi + J_\varphi \bigotimes J_\chi. \tag{3.2}$$

Here the first term in (3.2) is the free Hamiltonian of the first subsystem which must belong to the trivial (singlet) representation of $SU(3)$ (and I_χ is the unit operator acting in the subspace that describes the second subsystem), the second term is the free Hamiltonian for the second subsystem and the third term describes the colour interaction between the subsystems. The colour currents J_φ and J_χ must belong to some nontrivial representations of $SU(3)$, and their direct product must be constructed in such way as to belong to the singlet representation of the symmetry group. Consequently, the term $J_\varphi \bigotimes J_\chi$ contains more than one item [23]. Let us now act upon one branch $|\varphi_k\rangle|\chi_k\rangle$ from the decomposition (3.1) by the evolution operator determined by Hamiltonian (3.2). Due to the structure of the term $J_\varphi \bigotimes J_\chi$, including several items, our branch turns into several branches, similarly to the splitting of the branch $|\rightarrow\rangle|\Phi_\rightarrow\rangle$ in the Stern-Gerlach experiment under the action of the measurement operator $\hat{U}$ (see Sec. 2). Every branch $|\varphi_k\rangle|\chi_k\rangle$ will undergo this additional splitting and this phenomenon does not depend on the particular way of decomposition of the wave function (3.1). Independently of the choice of our preferred basis, its elements ("branches") will rotate with permanent velocity in the space of internal colour degrees of freedom. Thus we have no preferred basis for this decomposition, because we have no branches stable against the quantum evolution. All the above considerations remain true in the cases when the subsystem I is not a single quark but an arbitrary combination of quarks and antiquarks belonging to some non-singlet representation of $SU(3)$. Moreover, our arguments are applicable to any non-Abelian symmetry group. At the same time, a splitting of the system into singlet subsystems resolves the problem of stability of the preferred basis because in this case the non-trivial (colour) interaction between subsystems cannot take place. Thus we have seen that the preferred basis selection rules formulated in Sec. 2 can in some cases affect the choice of the system splitting into subsystems. The consistency requirement imposes some restrictions on the arbitrariness of this splitting. In the case of the $SU(3)$ colour group of quantum chromodynamics the impossibility of considering a single quark or some coloured combination of quarks and antiquarks as a subsystem can be treated as some hint upon a purely quantum-mechanical proof of the confinement phenomenon in QCD which is intensively discussed in the literature [24]. Let us stress once again: in the presence of an exact non-Abelian symmetry of the wave function of the Universe and the Hamiltonian governing its evolution, a decomposition of the Universe into non-singlet parts does not induce a unique and consistent choice of a preferred basis since all the possible bases will be unstable against an interaction Hamiltonian, belonging to the trivial representation of the corresponding non-Abelian group but composed from non-trivial currents. Note that, treating Stern-Gerlach experiment in Sec. 2, we could choose the unique preferred basis even in the case of a degenerate density matrix of the atom (the density matrix degeneracy is equivalent to the fact that the wave function is invariant under the action of the rotation group $SO(3)$) because the interaction Hamiltonian for the atom and the device was not invariant under $SO(3)$ and had a chosen direction, the magnetic field orientation.

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