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GRAVITATIONAL RADIATION, DARK MATTER AND INDUCED RESONANCE IN NEUTRINO SYSTEMS

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A phenomenological model, predicting resonance phenomena in neutrino flows propagating in the field of gravitational radiation in the presence of a substantial environment, is suggested.

1. Introduction

The integration of neutrino astronomy and gravitational wave astronomy represents an obvious tendency in the development of modern science of the Universe [1]. The interest in correlation analysis of temporal, spatial and spectral characteristics of photon, neutrino and graviton flows arising at supernova explosions [2,3] gives the most pronounced evidence to the fact. However, not only astrophysical and cosmological scenaria of neutrino and graviton flows origination are interesting from the theoretical point of view, but also the peculiarities of their interactions in the presence of a substantial environment in the course of their long-term joint propagation.

We would like to put forward a hypothesis that the presence of the third, namely, substantial, component strengthens in some cases the interaction between neutrinos and gravitons. The point is that, due to its nature, the gravitational radiation breaks down the equilibrium stationary state in the "neutrino + medium" system and initiates there the processes connected with neutrino exchange between the flow and the medium. No matter how weak may be the exchange process, under the condition that the propagation velocities of the individual massless neutrinos and gravitons are equal, it results in resonance phenomena, capable of distorting the spectral characteristics of the neutrino flow due to its long-term propagation from a source to an observer.

In order to confirm this hypothesis, we consider here an exactly integrable phenomenological evolutionary model of a neutrino system against a gravitational radiation background in the presence of a substantial environment, the latter being interstellar dust, dark matter or other distributed objects. For the sake of simplicity of presentation we will name this third component dark matter.

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2. Hierarchic model. Evolutionary equations

Let us consider a three-component hierarchic system with the following coordination scheme:

(i) Gravitaional Wave Background
The gravitaional wave background is described by the metric

$$ds^2 = dudv - A^2(u)dx^{2^2} - B^2(u)dx^{3^2};$$
$$u = ct - x^1; \quad v = ct + x^1;$$
$$A''/A + B''/B = 0, \tag{1}$$

which is an exact solution of Einstein's vacuum equations [4]. The intrinsic gravitational fields of dark matter and neutrinos, as well as their redistribution, do not cause noticeable changes of the gravitaional wave field.

(ii) Dark Matter
Dark matter is treated as a quasi-thermo-reservoir for neutrinos and evolves only under the influence of gravitational radiation field.

(iii) Neutrino System
The neutrino system is described by the general-relativistic kinetic equation [5]

$$p^i \frac{\partial f_\nu}{\partial x^i} - \Gamma^i_{kl} p^k p^l \frac{\partial f_\nu}{\partial p^i} + \frac{\partial}{\partial p^i} \left(Q f_\nu F^i \right) = 0. \tag{2}$$

Here p^i is the neutrino momentum 4-vector; it is a null vector ($g_{ik}p^ip^k = 0$) if the neutrinos are massless; $f_\nu(x,p)$ is the neutrino distribution function; Γ^i_{kl} are the Christoffel symbols; Q is a conformal factor; the vector field $F^i(x,p)$ models a force acting on the neutrinos from dark matter. We suggest that the Eq. (2) is collisionless, i.e., the neutrino-neutrino collision integral is negligibly small as compared with the force-like part. The vector F^i should be orthogonal to the momentum vector ($g_{ik}p^iF^k = 0$) due to a normalization condition [5].

The construction

$$F^i = G^i_{.k}p^k = (\lambda\delta^i_k + \omega^i_{.k})p^k, \quad (3)$$

identically satisfies the orthogonality condition on the null-rest-mass-shell in the phase space [5]. (Here $\lambda(x,p)$ is a scalar function; $\omega_{ik}(x,p)$ is an antisymmetric tensor ($\omega_{ik} = -\omega_{ki}$)). We suggest that in the absence of gravitational radiation λ and ω_{ik} are zero. The emergence of nonvanishing values of λ and ω_{ik} functions may be attributed to space-time curvature, as well as the appearance of gravitaional wave-induced nonequilibrium fluxes in dark matter.

3. Hierarchic system symmetry and a singular solution of the kinetic equation

The symmetry of the space-time surrounding the dark matter and the neutrino system, is predetermined by the symmetry of the gravitational radiation field. The space-time with the metric (1) admits a five-order group of motion with the following Killing vectors [4]:

$$\xi^i_{(1)} = \delta^i_v; \quad \xi^i_{(2)} = \delta^i_2; \quad \xi^i_{(3)} = \delta^i_3;$$
$$\xi^i_{(4)} = 2x^2\delta^i_v + D^{22}\delta^i_2; \quad \xi^i_{(5)} = 2x^3\delta^i_v + D^{33}\delta^i_3;$$
$$D^{\alpha\beta} = \int_0^u g^{\alpha\beta}(u')du' \qquad (\alpha,\beta = 2,\ 3). \quad (4)$$

Along $\xi^i_{(a)}$ $(a = 1,\ldots,5)$ the Lie derivatives of the metric coefficients are zero ($L_{\xi_{(a)}}g_{ik} = 0$). It is now possible to draw a number of conclusions concerning the construction of the force-like field F^i, without defining concretely the dark matter structure and taking into account only the reasons for symmetry.

Let us suppose that, in the course of the evolution, dark matter inherits the gravitational radiation field symmetry [6]. Being massive, the substantial medium, the dark matter, is able to inherit only the symmetry connected with the Killing vectors $\xi^i_{(1)}, \xi^i_{(2)}, \xi^i_{(3)}$, so that the equations

$$L_{\xi_{(a)}}\Psi^{\cdots}_{\cdots}(x) = 0, \quad (a = 1,2,3), \quad (5)$$

are fulfilled for all dynamical and informational characteristics $\Psi^{\cdots}_{\cdots}(x)$ with an arbitrary tensor structure. It is important to emphasize that the requirements of inheriting the symmetry connected with the Killing vectors $\xi^i_{(4)}, \xi^i_{(5)}$ leads, in particular, to the consequence that the dark matter macroscopic velocity vector must be null, thus contradicting its obligatory property to be time-like.

If the space-time symmetry is inherited by dark matter, the tensor G_{ik} may also possess the property $L_{\xi_{(a)}}G_{ik} = 0$ $(a = 1,2,3)$. In this case the scalar function λ and the tensor one ω_{ik} depend only on the retarded time u. The gravitational radiation field symmetry brings into consideration some new momentum variables:

$$P_v \equiv \xi^i_{(1)}P_i, \quad P_2 \equiv \xi^i_{(2)}P_i,$$
$$P_3 \equiv \xi^i_{(3)}P_i, \quad P_u \equiv \frac{1}{2}(P^0 + P^1) \quad (6)$$

instead of the momentum vector components P^i, The following normalization condition is used:

$$g_{ik}P^iP^k = 4P_uP_v + g^{22}(u)P_2^2 + g^{33}(u)P_3^2 = 0. \quad (7)$$

A singular solution of the kinetic equation

We use the term singular solution to designate the function

$$f^{\text{sing}}_\nu(x,p) = f_\nu(x,p)\delta(P_i\xi^i_{(1)})\delta(P_i\xi^i_{(2)})\delta(P_i\xi^i_{(3)}), \quad (8)$$

where $\delta(z)$ is the Dirac delta function. This function cannot reduce to the special case of the well-known covariant construction [5]

$$f_\nu(x,p) = f_\nu\delta(P^iP_i), \quad (9)$$

and possesses a unique property: all its macroscopic moments

$$M_{i_1i_2\ldots i_n}(x) = \int P_{i_1}\cdot P_{i_2}\cdots P_{i_n}\cdot f^{sing}_\nu dP$$
$$= \delta^u_{i_1}\cdot\delta^u_{i_2}\cdots\delta^u_{i_n}\cdot M_{(n)}(u,v) \quad (10)$$

may be Lie-constant along all five Killing vectors:

$$L_{\xi_{(a)}}M_{i_1\ldots i_n}(x) = 0, \quad (a = 1,2,3,4,5) \quad (11)$$

for the co-moving neutrino flows. The f^{sing}_ν function satisfies the kinetic equation if the following relation takes place:

$$\xi^i_{(a)}F_i = 0, \quad (a = 1,2,3). \quad (12)$$

Keeping in mind that on the null-rest-mass-shell the quantities $P_v,\ P_2,\ P_3$ vanish, we may rewrite Eq. (12) in the scalar-like form

$$G_{ik}\xi^i_{(a)}\xi^k_{(a)} = 0, \quad (a = 1,2,3), \quad (13)$$

(or explicitly: $\omega_{v_2} = \omega_{v_3} = 0$). After integration with respect to the variables $P_v,\ P_2,\ P_3$ the reduced function f_ν, depending only on $P_u,\ u,\ v$, may be obtained from the characteristic equations

$$\frac{du}{0} = \frac{dv}{2P_u} = \frac{dP_u}{2QP_u(\lambda g_{uv} + \omega_{uv})} = -\frac{df}{QfD}, \quad (14)$$

where $D = (\partial F^i/\partial P^i)|_{P_v=P_2=P_3=0}$.

If the functions λ and ω_{ik} do not depend on the vector P^i, we obtain from (14) that

$$P_u = \Pi + Qv(\tfrac{1}{2}\lambda + \omega_{uv}); \quad f_\nu = f^0_\nu(\Pi)\tfrac{\Pi}{P_u};$$
$$\Pi = \text{const} = P_u(v = 0). \quad (15)$$

It is of interest to consider another example, when $\omega_{uv} = 0$ but λ depends linearly on the momentum P^i:

$$\lambda = \lambda^0(u)(P_i U^i), \tag{16}$$

where $U^i(u)$ is the macroscopic velocity of dark matter as a whole. In this case we obtain

$$P_u = \Pi \exp[\lambda^0(u) v Q(U_i \xi^i_{(1)})]; \quad f_\nu = f^0(\Pi)\frac{\Pi}{P_u}; \quad \Pi = P_u(v=0) = \text{const}. \tag{17}$$

4. Macroscopic properties of the singular solution

The following conclusions from Eqs. (15)(17) are obvious:

1. If the influence of dark matter on the neutrino system is such that $\frac{1}{2}\lambda + \omega_{uv} = 0$ (in particular, $\lambda = 0, \omega_{uv} = 0$), P_u is constant and the energy of an individual neutrino $e = c(P^i U_i) = 2cP_u U_v,$, moving along geodesic lines in the gravitaional wave field ($U_v = \text{const}$), calculated in the dark matter frame of reference, is constant; the particle co-moves with the gravitaional wave front.

2. If the relation $\frac{1}{2}\lambda + \omega_{uv} \neq 0$ is valid, the neutrino energy changes with increasing advanced time v, the change being linear when the solutions (15) are used and exponential in the case of the solution (17).

3. All macroscopic moments of the distribution function f_ν^{sing} have the common structure (10), where

$$M_{(n)}(u,v) = \frac{1}{AB}\int_0^\infty \Pi \, d\Pi f^0(\Pi)(P_u)^{n-1}. \tag{18}$$

It follows from Eqs. (10) and (18) that the moments of the second and higher orders grow with increasing v if $\frac{1}{2}\lambda + \omega_{uv} \neq 0$, the growth being polynomial or exponential if P_u is defined according to (15) or (17), respectively.

4. In the special case $n = 2$, when the moment $M_{i_1 i_2}(x)$ is proportional to the neutrino energy-momentum tensor $T_{i_1 i_2}(x)$, the energy density of the neutrino system Θ is seen to increase from the point of view of any geodesic observer:

$$\Theta = T_{ik}U^i U^k = 4T_{uu}U_v^2. \tag{19}$$

If $f^0(\Pi)$ is a Fermi-Dirac function

$$f^0(\Pi) = (2\pi\hbar)^{-3}\left[\exp\frac{c\Pi}{k_B T} + 1\right]^{-1}, \tag{20}$$

and $U_v = U_u = 1/2$ (the obsever is at rest), the relative energy deviation in the case (15)

$$\frac{\Theta - \Theta(v=0)}{\Theta(v=0)} = Qv(\frac{\lambda}{2} + \omega_{uv})\frac{\int_0^\infty \Pi d\Pi f^0(\Pi)}{\int_0^\infty \Pi^2 d\Pi f^0(\Pi)} = Qv(\frac{\lambda}{2} + \omega_{uv})\frac{c}{k_B T}\frac{\zeta(2)}{3\zeta(3)} \tag{21}$$

grows linearly with increasing v and depends on the temperature T of the neutrino system ($\zeta(N)$ is the Riemann zeta function). Similar calculations in the case of Eq. (17) give the temperature-independent result

$$\frac{\Theta - \Theta(v=0)}{\Theta(v=0)} = \exp[Q\lambda^0(u) v U_v] - 1. \tag{22}$$

5. The conclusion that we deal with resonance phenomena, able in principle to distort the neutrino spectrum, is evident.

5. Conclusions

The suggested model foretells that under the influence of the force-like effect from dark matter, resonance phenomena induced by the gravitational radiation field become possible in a neutrino system, so that a search for their astrophysical and cosmological consequences seems to be reasonable. The authors forecast at least two potential applications of the idea. The first one is a periodic change of the solar neutrino spectrum caused by periodic screening of gravitational radiation sources by the Sun; and the second one is an anisotropy of the neutrino relic background attached to the directions of gravitational radiation sources.

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