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RELATIVISTIC PHYSICS AND GEOMETRY

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The problem of geometrization of physics is considered as a part of problems contained in Hilbert's sixth problem. This Hilbert's problem concerns mathematical formulation of physics axioms. It is shown that for the whole 20th century this problem has been forming the scientific research strategies in theoretical physics and some areas of mathematics, especially in geometry. The appearance of special and general relativity as well as geometric gauge field theory can be regarded as consecutive stages in the solution of Hilbert's sixth problem. A present-day problem consists in application of the geometric gauge field theory to relativistic nuclear physics.

Релятивистская физика и геометрия
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Проблема геометризации физики рассматривается как часть шестой проблемы Гильберта. Эта проблема Гильберта касается математической формулировки аксиом физики. Показано, что в течение всего XX века данная проблема формировала стратегии научных исследований в теоретической физике и некоторых разделах математики, особенно в геометрии. Появление специальной и общей теорий относительности, как и геометрической теории калибровочных полей, можно рассматривать как последовательные стадии решения шестой проблемы Гильберта. Проблемой сегодняшнего дня является применение геометрической теории калибровочных полей к релятивистской ядерной физике.

1. Hilbert's Problem VI

As is known, in 1900, D. Hilbert formulated 23 problems which, in his opinion, ought to be solved by mathematicians of the 20th century [1]. Among them, the sixth problem has pointed at the necessity to state the mathematical formulation of physics axioms. In this connection, Hilbert proposed to construct physical axioms like the axioms of geometry. So, Hilbert's Problem VI contains the problem of geometrization of physics as its part.

For the whole 20th century, this problem has been forming strategies of scientific research in theoretical physics and in different mathematical areas, especially in geometry. The appearance of special (SR) and general (GR) relativities as well as geometric gauge field theory can be regarded as consecutive stages in the solution of Hilbert's Problem VI [2,3].

In connection with new physical theories, the corresponding new geometries were appearing. New mathematics stimulated the development of physicals and vice versa. Minkowski's 4D geometry was created for SR, Cartan's formulation of Riemannian 4D geometry arose in GR. At last, the fibre bundle space geometry was formulated as an extension of Cartan's geometry. It was used by me for a geometric formulation of gauge field theory ([4]). At present, this theory is the greatest extension of GR. Gauge field theory proved to be very successful in explanation of phenomena of particle physics and gravity. It permits the construction of a unified theory of all fundamental interactions. Moreover, such a theory can be formulated in both usual and geometric forms.

The problem, at which Einstein had been working for many years, is solved today owing to geometric gauge field theory.

2. The way to geometric relativity

In one of his papers, Einstein explained why he decided to seek a way to a geometric form of gravitation theory. He called his predecessors on this way I. Kant, German philosopher of 18th century, and A. Poincaré, French mathematician of the 19th century.

Kant [5] established that any experimental description consists of two parts: geometry (or coordinates) and forces. At the same time, it is known that force-free or inertial motions do exist, but a geometry-free motion cannot exist. In an experimental description, any point, particle or event are supplied with coordinates. But forces can be absent.

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Poincaré [2] suggested that one can always find such a geometry that makes any motion look force-free. This idea stimulated Einstein to seek a geometric description of particle motion in a gravitational field.

This situation can be represented by the symbolic formula

$$G = G_0 + F, \tag{1}$$

where G_0 is a background geometry, i.e., a rigidly given one; F is an image of acting forces; G is a dynamical geometry in the Einsteinian sense, i.e., it changes according to particle motion.

It is necessary to note that here the corresponding equations are the same in the left- and right-hand sides of Eq. (1).

Since the equality sign has only a symbolic sense, the variable sets in the left- and right-hand sides can differ from each other. Poincaré assumed that the choice of variables depends on a convention among scientists. Consequently, one can use the geometry which is more suitable for calculations, and so any geometry can arise and be applied for finding a solution of the equations.

Einstein decided to write the equations which describe particle motion in a gravitational field as a free motion. As is known, he chose Riemannian geometry to solve this problem and obtained the equation of a geodesic line [6]).

But, in addition, Einstein decided to clarify the physical sense of variables in his equations. He ascertained that his equations of motion described not any particle motion but only that of particles having some special properties. He named such particles test bodies. They were subject to gravitational influence of an external field, but were not to change this field, i.e., had no back influence. In this way, Einstein demonstrated that a modification of the set of variables in equations implies a change in the physical properties of the objects or experimental conditions which are described by these equations.

3. Innateness of geometry, conventionalism and overcoming them

A key problem in the interpretation of the Poincaré-Einstein symbolic formula (1) is: where does geometry come from?

Kant assumed that geometry is innate and arises at the same time when a child has been born. Such an answer was unacceptable for many scientists and philosophers. In spite of Kant's point of view seeming strange, he proved to be partly right. Scientific investigations of French physiologists of the 19th century showed that the human equilibrium organ consists of three almost mutually perpendicular planes. As a result, a human can differ three spatial dimensions at the time of his birth.

In contrast to Kant, we have to note that in reality a human being has only innate organs for getting geometric knowledge, but not this knowledge itself. His body is a natural coordinate system and instrument for geometric constructions. But it does not contain Euclidean theorems in itself. Means of getting knowledge and the knowledge itself are not one and the same.

Poincaré's point of view consisted in the following: separation of the right-hand side into two parts, $(G_0 + F)$, depends on us, and it is subject to convention.

This statement is known as conventionalism and was often severely criticized by materialist philosophers. But here is a real way out. When we try to apply the equations to real objects' behaviour in the experiment, it will be clear that G_0 is a mathematical image of the device realizing the coordinate system. Thus, in practice, the freedom of choosing G_0 becomes the freedom of choosing instruments for coordinate system construction. This choice really depends on us, but it is formed by the experimental conditions rather than a convention among scientists. So, just like Kant, Poincaré was right only partly.

All questions connected with the Poincaré-Einstein formula received the most serious study in the papers by N.P. Konopleva and H.A. Sokolik in the 1960s (see [7] etc.). These papers cover the problems of the sovereign physical theory structure.

4. Geometry in the structure of physical theory

A physical theory is named sovereign if it has its own means for distinguishing true and false conclusions, and consequently does not need an experiment for solving such problems. Only a sovereign theory conclusions can be regarded as the truth. The conditions which a sovereign physical theory should satisfy were investigated in Ref. [8].

The structure of axioms of such a physical theory must reflect the specific way in which information on the external world comes into the theory. As is known, the data of a physical theory come from experiments. Therefore the structure of axioms of a sovereign physical theory must be closely connected with the principles of experimental investigations.

For instance, the result reproducibility requirement leads to the fact that the language of theoretical physics must be Lie group theory. The symmetry properties of a theory become determining ones. If the symmetry is global, we should use finite Lie groups. If the symmetry is local, we should use infinite Lie groups. By means of representations of finite Lie groups, the classification of elementary particles was obtained. From representations of infinite Lie groups, there appeared classification of elementary particle interactions. A Lagrangian theory of gauge fields on the base of infinite Lie group representations was constructed by N.P. Konopleva in 1967 [9].

Among the gauge fields, gravity is associated with the group of local translations, which are usually called generally covariant coordinate translations in 4D Riemannian space-time. The above Lagrangian formalism and consideration of the coordinate translation group as a local gauge group permits one to obtain the Einsteinian theory of gravitation as the theory of one of the gauge fields. This is the only way of obtaining usual GR as a gauge field theory. Torsions are absent in this approach.

In gauge field theory, all non-gravitational fields are described by nonlinear extensions of Maxwell's equations. Electrodynamics equations coincide with Maxwell's equations.

When fundamental interactions are considered in Riemannian space-time, the Einsteinian equations must be added to the other equations of the theory as local relativistic vacuum equations [10]). Thus the physical sense of the Einstein equations becomes wider.

It is remarkable that in this scheme the equations of particle motion can be obtained by differentiation of the field equations. It is the same situation that we have in GR. In principle, the geodesic equations could be eliminated from the GR axioms because of this fact. On the other hand, the paths of all particles carrying the corresponding gauge charge in the external gauge field look like paths of test bodies. Therefore the Lorentz equation describing the motion of an electron in an external electromagnetic field turns into the equations of electromagnetic test body motion.

When only free electromagnetic and gravitational fields are present, the set of equations of gauge field theory coincides with the Wheeler-Misner equations of geometrodynamics [11]).

So, let us return to geometric axioms in relativistic physics.

Einstein explained the essence of his geometrical approach by an imaginary experimenter located in a falling lift. This experimenter has a ruler and a clock, which permit him to measure segments in space and time periods. Therefore he can construct a local coordinate frame in his neighbourhood. This coordinate frame will be a basis of a local Euclidean space in the falling lift. The origin of this coordinate frame will coincide with a test body freely falling in the external gravitational field. The equivalence principle, which is one of the GR axioms, states that in this situation the experimenter does not feel any gravitational field influence. Near him, all events happen in just the same way as in the absence of gravity. At the same time, another experimenter, located on the Earth's surface, outside the lift, will interpret the first experimenter's motion as noninertial motion in the gravity field of the Earth. Both these descriptions are correct, but the first one corresponds to a local description of motion in the accompanying coordinate system, while the second one uses the global Cartesian coordinate system associated with the Earth. In the second case, gravity describes forces acting in global Euclidean space. This is one of the realizations of the Poincaré-Einstein formula. The equality sign corresponds to the equivalence principle. Two descriptions can be brought into accord with each other by identification of gravity forces with connection coefficients of 4D Riemannian space-time.

Can this method be extended to other interactions?

For 30 years after creation of GR, Einstein tried to unify geometrically gravity and electromagnetism. Many other authors did the same as Einstein. But at that time geometry had not yet any means for solving this problem. Cartan's formulation of Riemannian geometry of 1925 [12]) adequately described the falling lift situation but was insufficient for the new task.

Only in the 60s of the 20th century the fibre bundle space geometry became sufficiently developed for its application to physics.

In 1964, it occurred to me how the Einsteinian problem could be solved [13]). To this end, one should answer the question: what is a mathematical image of other measuring devices besides rulers and clocks used by the experimenter in a falling lift? My answer was the following. A mathematical image of any device in physical theory is the space of parameters measured by this device. In this space, some coordinate frame can be chosen as in usual space. Its origin must coincide with the origin of usual spatial coordinates in which the experimenter works in the falling lift. It means that this experimenter has not only rulers and clocks, but also a voltmeter etc. Mathematically it leads to an increase in the dimensions of space at the point where the experimenter is located. At the same time, in the opinion of an external observer, the experimenter moves as before in usual 4D space-time. Thus our problem reduces to carrying some multidimensional space along lines in 4D space-time.

Such a procedure was unknown in theoretical physics. But what could be said about it by mathematicians? And I went to the faculty of mathematics and mechanics ("Mechmath") of the Moscow State University.

In its library, I found G.F. Laptev's thesis of 1952 on embedded manifolds [14]. Then I learned that he was leading a seminar on this problem at Prof. P.K. Rashevski's chair of Higher Geometry at the Mechmath. As it turned out, the geometry which I was looking for did not yet exist but was just arising before my eyes. Now it is called fibre bundle space geometry.

I began to attend Laptev's seminar and took part in all scientific conferences on differential geometry held in the USSR at that time. My talks were put in sections of geometry applications. Unfortunately, these conferences had almost no proceedings. But I also told on application of fibre bundle geometry in physics at conferences on theoretical physics, elementary particles and gravity, as well as on philosophy and science methodology. My philosophical and methodological papers were published together with H.A. Sokolik. I completed a geometric formulation of gauge field theory in 1967 and reported it at the corresponding conferences in Kazan [15] and Tbilisi. Then in 1969 I defended PhD thesis entitled "Geometric Description of Interactions" at Lebedev Physical Institute of the Academy of Sciences of the USSR and, by invitation of Prof. A.M. Baldin, reported its results at the International Seminar on Vector Mesons and Electromagnetic Interactions at JINR, Dubna [16]. My thesis was written without any use of post-graduate course. It was recommended to be published by the Academic Council of Lebedev Physical Institute. In 1972, our book with V.N. Popov, "Gauge Fields", was published in Russian by Atomizdat. For this book, V.N. Popov wrote Chapter IV on gauge field quantization with path integrals [17].

After that, the geometrical treatment of gauge fields in terms of fibre bundle space geometry [18]) became generally recognized and induced the development of superspace geometry in mathematics and supersymmetric gauge theories in physics. The Kaluza-Klein [19] and Weyl [20] theories attracted the physicists' attention again.

5. Conclusions

So, where have we arrived by axiomatization and geometrization of physics according to Hilbert's Problem VI?

Unification of electrodynamics and mechanics led to creation of SR in physics and 4D Minkowski geometry in mathematics. Finite Lie groups found their wide application in physics, especially in quantum mechanics and elementary particle physics. They became the basis of classifications of elementary particles, atomic and nuclear states.

But, for a long time, infinite Lie groups could not find their place in physics. The appearance of local coordinate translations in GR induced doubt about the physical sense of this theory. Later, the same doubt appeared about gauge field theory using local gauge symmetry groups. These groups belong to infinite Lie groups similarly to local coordinate translations in GR.

The point is that finite Lie groups have invariants, whereas infinite Lie groups have none. Therefore usual conservation laws vanish when the symmetry of the theory becomes local. The dynamical constants are just the numbers which a physical theory produces to compare them with experimental data. Without conservation laws, we cannot construct dynamical constants for experimental description.

But it occurred to me that local symmetries should not be used for obtaining dynamical constants. In Utiyama's opinion [21], they must classify interactions between particles, but not these particles themselves. Our (with Sokolik) point of view consisted in that local symmetries ensure the existence of gauge fields. They generate the appearance of connection coefficients in a space which is only locally homogeneous. Connection coefficients are geometric objects. What kind of physical objects correspond with them?

Einstein tried to geometrize electrodynamics using additional metric coefficients. Such coefficients arose when the space-time dimension increased (the Kaluza-Klein approach) or the 4D space-time metric became asymmetric. In these cases, the vector potentials of the electromagnetic field became components of the metric tensor.

Weyl was the first who identified the electromagnetic vector potentials with connection coefficients, but it was only possible in terms of a new geometry which Weyl constructed for this task (known as the Weyl geometry [20]). Unfortunately, in this geometry, a correct description of Einsteinian gravity became impossible.

The problem was solved when the fibre bundle space geometry arose. I identified the gauge field vector potentials with the connection coefficients of fibre bundle space. 4D Riemannian space-time turned into the base of the fibre bundle space, and the space where local gauge groups were acting was identified with a fibre of the fibre bundle space. Gravity and nongravitational interactions became untied. Now they became acting in different spaces: gravity existed in the base while nongravitational fields were acting between fibres of the fibre bundle space. GR, SR, Maxwell's electrodynamics, the Wheeler-Misner geometrodynamics, the Yang-Mills equations [22]) were exactly reproduced in this geometry terms. Moreover, the way of unifying all interactions in both usual and geometric form was opened.

Therefore I continued my work in spite of sharp criticism with respect to local gauge theories from some well-known scientists (V.A. Fock [23], V.I. Ogiyevetsky [24], E.S. Fradkin [25], B.L. Ioffe etc.). This skepticism was overcome by the creation and use of new mathematical methods in both mathematics and physics. It has been the above-mentioned Lagrangian formalism for infinite Lie groups (1967, N.P. Konopleva), fibre bundle space geometry (Soviet and foreign mathematicians), geometric interpretation of gauge fields in terms of the new geometry (1967, N.P. Konopleva), and quantization of gauge fields by path integrals performed in 1967 by B. DeWitt [26]), L.D. Faddeev and V.N. Popov [27]). Renormalization of Yang-Mills fields was performed in 1971 by J.C. Taylor [28]) and in 1972 by A.A. Slavnov [29]) (the massless case), and by G. 't Hooft [30]) (massive case) in 1971. Discussions on fundamental questions of quantum gauge field theory can be find in [31]).

The quark models of elementary particles [32, 33] appeared in 1964 [34]). They played a very important part in the process of gauge field theory application to particle physics. Just these models have proved the correctness of gauge field theory in its usual form in Minkowski space-time. Today the corresponding unified model of fundamental interactions is known as the Standard Model.

Next in turn should be a verification of gauge field theory in its geometric form. Such experiments will be analogous to GR experiments, and now they seem very complex.

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